Adword auctions: impact of ranking strategies and competition on search engines revenues

Bruno Tuffin (collaboration with Patrick Maillé)

Inria Rennes - Centre Bretagne Atlantique

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Outline

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3. On the ranking strategy in adword auctions
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   - Model
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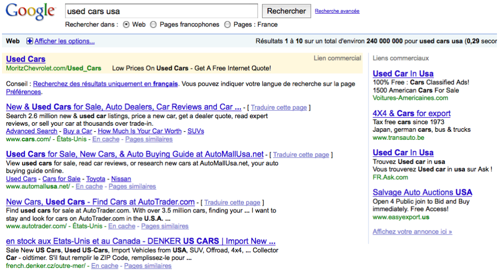
Introduction to adword auctions

- Search engines play a crucial role in the Internet.
- Revenue through advertising slots, usually displayed at the top or right of the search page.
- Advertisers submit bids for relevant keywords only.



- Allocation of slots thanks to adword auctions.
  - combined revenue of Yahoo! and Google in 2005: \$11 billion in 2005
  - expected to count for 40% of total advertising revenue.

Auction principle (single keyword, K slots)

- Advertisers submit bids for specific keywords.
- Each time there is a search on that keyword:
  - advertisers are ranked and allocated slots according to a prespecified criterion:
    - bid value (initially for Yahoo!)
    - the revenue they will generate (more or less Google).
  - Possible payment rules:
    - *Pay-Per-Impression* (PPI): advertisers charged every time their ad is displayed
    - *Pay-Per-Click* (PPC): advertisers is charged only when the ad is clicked
    - *Pay-Per-Transaction* (PPT): advertisers charged when a sell.
  - Amount to be paid each time?
    - First Price: advertisers pay their bid
    - Generalized Second Price (GSP): they pay the bid of advertiser below them in the ranking
    - Vickrey-Clarke-Groves (VCG) auctions: you pay the opportunity cost that your presence introduce to all other advertisers.
- In use: PPC and GSP. But bid-based or revenue-based ranking?

Goal(s) of our work

We are focusing on the ranking strategies of search engines.

Considered issues:

- To see if a random ranking policy would not increase the revenue, when users potentially run the search several times, at different moments.
- To discuss the relevance of always prefering revenue-based ranking over bid-based.
- To investigate the best ranking strategy of search engines in competition (limited existing works) thanks to a two-levels game:
  - Largest time scale: search egines choose their ranking strategy (maximizing) their revenue
  - Smallest time scale: advertisers in competition for the advertising slots (by splitting their avertisement budget over the engines)

Outline

On the interest of introducing randomness in ad-word auctions

Maillé and T., 2010

A specific issue: users may compose the same keyword several times

- This situation may happen when users
  - do not remember the results
  - or require new or additional informations
- Traditional adword auctions will *always* display the same advertisers.
- But is it the most relevant procedure ?
  - If the ad not clicked through once, why always presenting it again?
- We propose to illustrate the potential benefits of using a

  random allocation rule.

Model

Simplified model to illustrate the phenomenon.

- A search engine providing only one commercial slot
- Two advertisers, say, 1 and 2, competing for that slot on a given adword:
  - b_i bid of advertiser i for that keyword,
  - π_i the probability that advertiser i's ad is displayed (which should depend on the bid profile (b_1, b_2)),
  - p_i the price-per-click that advertiser i is charged, also dependent on (b_1, b_2).

In a first step, we fix the bids and prices, and investigate the gain produced by a random assignment.

User behavior model

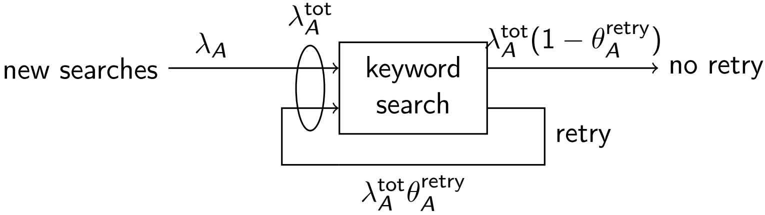
- Heterogeneous population:
  - type-A users only be interested in purchasing the good sold by advertiser 1, but can potentially click on the ad of advertiser 2 without purchasing it eventually.
  - type-B users behaving symmetrically with respect to advertiser 2.
- λ_A (resp. λ_B) average number of *first* requests per time unit of type-A (resp. type-B) customers.
- Click propability of a type-A (resp. B) customer if ad of advertiser i displayed: $c_{A,i}$ (resp. $c_{B,i}$).
- Purchasing probabilities $h_{A,1}, h_{B,2} > 0$ after clicking on the (corresponding) ad.
- Buying probabilities respectively for type-A and type-B customers:

$$\pi_1 c_{A,1} h_{A,1} \quad \text{and} \quad \pi_2 c_{B,2} h_{B,2}.$$

Customers not buying will retry later

- R_j probability of a not-buying type-$j \in \{A, B\}$ user to perform again the search later.
- For type-A: θ_A^{retry} the overall probability of retry, given by

$$\theta_A^{\text{retry}} := R_A(1 - \pi_1 c_{A,1} h_{A,1}).$$



- From $\lambda_A^{\text{tot}} = \lambda_A + \theta_A^{\text{retry}}\lambda_A^{\text{tot}}$,

$$\lambda_A^{\text{tot}} = \frac{\lambda_A}{1 - \theta_A^{\text{retry}}} = \frac{\lambda_A}{1 - R_A(1 - \pi_1 c_{A,1} h_{A,1})}.$$

- Similarly, $\lambda_B^{\text{tot}} = \frac{\lambda_B}{1 - R_B(1 - \pi_2 c_{B,2} h_{B,2})}$.

Search engine revenue

- Mean revenue per unit of time (with $\pi_1 + \pi_2 = 1$) :

$$\begin{aligned} U(\pi_1) &= p_1 \underbrace{\pi_1(\lambda_A^{\text{tot}} c_{A,1} + \lambda_B^{\text{tot}} c_{B,1})}_{\text{nb of clicks on ad 1}} + p_2 \underbrace{\pi_2(\lambda_B^{\text{tot}} c_{B,2} + \lambda_A^{\text{tot}} c_{A,2})}_{\text{nb of clicks on ad 2}} \\ &= \lambda_A \frac{\pi_1(p_1 c_{A,1} - p_2 c_{A,2}) + p_2 c_{A,2}}{1 - R_A(1 - \pi_1 c_{A,1} h_{A,1})} + \lambda_B \frac{\pi_1(p_1 c_{B,1} - p_2 c_{B,2}) + p_2 c_{B,2}}{1 - R_B(1 - c_{B,2} h_{B,2} + \pi_1 c_{B,2} h_{B,2})} \end{aligned}$$

- **Proposition:** there exists a unique π_1^* maximizing the revenue $U(\pi_1)$ of the search engine. The solution is in the interior of the interval $[0, 1]$ if $U'(0) > 0$ and $U'(1) < 0$.
- **Example:** $c = 1/2$, $h = 1/2$, $\lambda_A = 1$, $\lambda_B = 0.8$, $p_1 = 1$, $p_2 = 0.8$.
  - With retry probability $R = 0.8$, the revenue is maximized at $\pi_1^* = 2/3$, and given by 1.4.
  - Compared with the optimal revenue when only one ad is displayed, $\max(\lambda_A p_1, \lambda_B p_2) = 1$, a gain of 40% is observed.

Game between advertisers

- The previous analysis was for fixed bids and prices. But advertisers can play (i.e., submit bids representing how much they accept to pay) strategically according to the display probability.
- In the symmetric case (the asymmetric case handled similarly), mean sale incomes per unit of time (if v_i benefit per sale):

$$
\begin{aligned}
V_1(\pi_1) &= \underbrace{\lambda_A^{\text{tot}}\pi_1 ch}_{\text{sales per time unit}} \quad v_1 = \frac{\lambda_A \pi_1 ch}{1 - R(1-\pi_1 ch)} v_1 \\
V_2(\pi_2) &= \frac{\lambda_B \pi_2 ch}{1 - R(1-\pi_2 ch)} v_2.
\end{aligned}
$$

- Utilities

$$U_i = V_i(\pi_i) - p_i.$$

VCG auction

- Mechanism known to be *incentive compatible*, *individually rational* and *efficient* by maximizing social welfare.
- **Allocation rule** from the search engine: solve strictly convex optimization problem

$$\max_{\pi_1,\pi_2 \text{ s.t. } \pi_1+\pi_2\leq 1} \bar{V}_1(\pi_1) + \bar{V}_2(\pi_2),$$

  where $\bar{V}_i$ is the declared willingness-to-pay function of advertiser i ($\bar{V}_i = V_i$ if i bids truthfully).
- Solution known from the previous propositions.

VCG pricing rule

- Charged the loss of value each advertiser imposes on the other through its presence.
- Total price t_i per time unit that each advertiser i is charged under the VCG rule is given by

$$\begin{cases} t_1 = \bar{V}_2(1) - \bar{V}_2(\bar{\pi}_2) \\ t_2 = \bar{V}_1(1) - \bar{V}_1(\bar{\pi}_1). \end{cases}$$

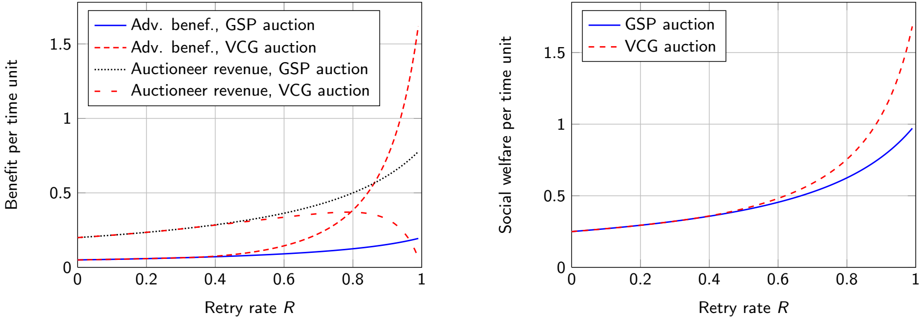
- If converted as a price per click

$$p_1 = \left(\bar{V}_2(1) - \bar{V}_2(\bar{\pi}_2)\right) \frac{1 - R(1 - \bar{\pi}_1 ch)}{\lambda_A \bar{\pi}_1 c},$$

$$p_2 = \left(\bar{V}_1(1) - \bar{V}_1(\bar{\pi}_1)\right) \frac{1 - R(1 - \bar{\pi}_2 ch)}{\lambda_A \bar{\pi}_2 c}.$$

Comparison with deterministic GSP

Let $v_1 = 1$, $v_2 = 0.8$, $\lambda_A = \lambda_B = 1$, $c = h = 0.5$ and investigate the influence of R.



- Revenue of advertisers larger with VCG on display probability, but not the engine revenue
- In the case of competitive engines, the one applying VCG is likely to be prefered.
- Social welfare better with VCG. The difference increases with R.

Outline

Comparing revenue-based and bid-based schemes

Maillé and T., 2011

- Most search engines have chosen (or switched to) a revenue-based ranking and charging;
- Is it always relevant?
- What we are going to show:
  - it depends on the parameters
  - but bid-based *can* be better than revenue-based.
  - Therefore: this has to be studied before making a choice or a move.
  - Two engines may have different optimal strategies when the CTRs of advertisers differ from an engine to another.

Basic model

- n advertisers submit bids to search engines for a given keyword.
- Only one slot (then GSP is VCG and truthful-bidding)
- The search engine apply Pay-Per-Click and GSP
- For advertiser i
  - CTR q_i taken from two classes of advertisers:
    - high-quality advertisers, with CTR q_h
    - low-quality advertisers, with CTR q_l, such that $q_l < q_h$.
  - The probability α of being a high quality one.
  - valuation/bid v_i random, distributed according to cdf F and density f, independent of the quality.
- For bid-based Pay-Per-Click GSP:
  - winner: highest bidder $i_b := \arg\max_i v_i$,
  - charged the second-highest bid $\max_{i \neq i_b} v_i$ at each click
  - revenue $q_{i_b} \max_{i \neq i_b} v_i$.
- For revenue-based Pay-Per-Click GSP:
  - winner: $i_r := \arg\max_i q_i v_i$,
  - charged per click: lowest p_{i_r} he could have bid to win $p_{i_r} = \frac{1}{q_{i_r}} \max_{i \neq i_r} q_i v_i$
  - revenue $q_{i_r} p_{i_r} = \max_{i \neq i_r} q_i v_i$.

Illustration: revenue-based pricing does not always yield a larger revenue

- $n = 2$ advertisers
- advertiser 1 has valuation $v_1 = 0.7$ and CTR $q_h = 0.6$
- advertiser 2 has valuation $v_2 = 0.5$ and CTR $q_l = 0.1$.
- With bid-based ranking rule
  - winner: advertiser 1
  - revenue will be $v_2 q_h = 0.3$.
- With revenue-based ranking rule
  - advertiser 1 still the winner $(v_1 q_h > v_2 q_l)$
  - price per click is $v_2 q_l / q_h$
  - revenue $v_2 q_l = 0.05$.
- Bid-based better.
- Intuition: second-ranked bidder may be a low-quality advertiser, resulting in smaller price per click when using the revenue-based scheme.

Average revenues

Proposition

The average revenue under bid-based ranking and charging is

$$R_b = n(n-1)(\alpha q_h + (1-\alpha)q_l) \int x(F(x))^{n-2}(1-F(x))f(x)dx.$$

Proposition

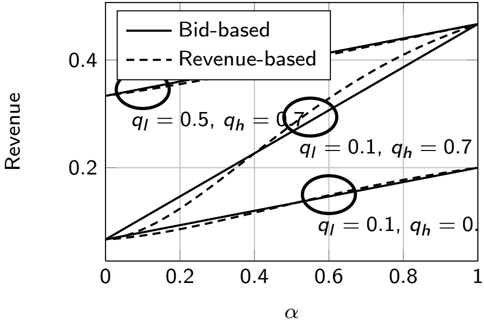
Define

$$\begin{aligned} G(x) &= \alpha F(x/q_h) + (1-\alpha)F(x/q_l) \\ g(x) &= \frac{\alpha}{q_h} f(x/q_h) + \frac{1-\alpha}{q_l} f(x/q_l). \end{aligned}$$

The average revenue under revenue-based ranking and charging is

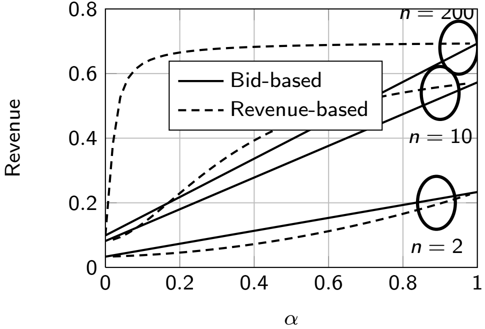
$$R_r = n(n-1) \int x(G(x))^{n-2}(1-G(x))g(x)dx.$$

Revenue in terms of the proportion α of high-quality users



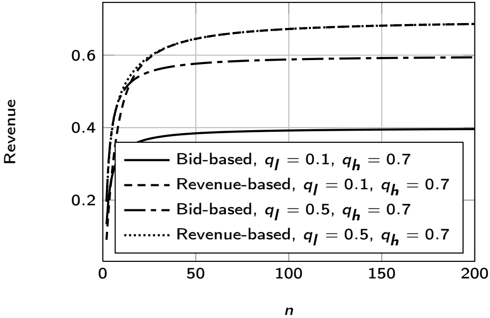
- With $F(x) = x$ for $x \in [0, 1]$
- For low values of α, and low values of q_l, bid-based ranking may produce a larger revenue
- Revenues (obviously) increase with the proportion of high-quality advertisers, and with the CTRs.

Revenues in terms of α for different values of n when $q_l = 0.1$, $q_h = 0.7$



- For a small number of advertisers the chances to get a larger revenue with bid-based ranking is larger (ex: $n = 2$).
- Less likely as the number of advertisers increases (never for $n = 200$).
- Reason: more chances to have a high-quality advertiser in second position with the revenue-based rule.

Revenues in terms of n for different values of q_l, q_h, when $\alpha = 0.5$



- As n increases, revenue-based rule yields a larger revenue.
- Average revenue asymptotically depends on the high-quality advertisers parameters with revenue-based (the two curves tend to coincide) because the second ranked advertisers tend to be a high-quality one.
- Not true for bid-based ranking rule because the valuation is independent of the quality parameter.

Outline

Analysis of competition among search engines

Maillé and T., 2011

- Two search engines (SE), labelled 1 and 2
  - a single advertisement slot at each SE
  - considering a single keyword, with λ, average number of searches per unit of time
  - α: (fixed) market share of SE 1 ($\alpha\lambda$ searches on SE 1).
  - Both SE apply GSP charging scheme.
- k advertisers:
  - budget b, taken from cdf $G(b)$
  - valuation per click v, taken from the cdf $F(v)$
  - Click-Through-Rate (CTR) *separable*, as the product of the CTR of the search engine, q_1 and q_2 for SE 1 and 2 respectively, and of the CTR c_i of the considered Advertiser i.
  - Goal of advertiser i: split their budget b_i over the two SEs: β_i on SE 1 and $1-\beta_i$ on SE 2.
  - Remark: advertisers' interest is to bid their valuation v_i since GSP is VCG when a single slot.

Key parameters and advertisers utilities

- Key performance parameters:
  - $w_j(v_i)$: probability that i wins on SE j given that her bid/valuation is v_i
  - $\mathbb{E}[R_j|v]$ being the average price paid on SE j *having won with* v.
  - $p_i^{(j)}$: probability that advertiser i submits a bid on SE j.
- Those parameters can be computed for both bid-based and revenue-based rankings, by solving sets of equations.
- Revenues, for $\beta = (\beta_1, \ldots, \beta_k)$ profile of strategies of advertisers:

$$U_i(\beta) = q_1 c_i w_1(v_i)\lambda_i^{(1)}(v_i - \mathbb{E}[R_1|v_i])q_2 c_i w_2(v_i)\lambda_i^{(2)}(v_i - \mathbb{E}[R_2|v_i]).$$

Game between advertisers on budget repartition at each SE

We consider two SEs and two advertisers.

- The two advertisers play, trying to (selfishly) maximize their own utility/revenue.
- Equilibrium notion, Nash equilibrium: profile of proportion strategies (β_1^*, β_2^*) st $\forall \beta_1, \beta_2 \in [0, 1]$,

$$U_1(\beta_1^*, \beta_2^*) \geq U_1(\beta_1, \beta_2^*) \text{ and } U_2(\beta_1^*, \beta_2^*) \geq U_2(\beta_1^*, \beta_2).$$

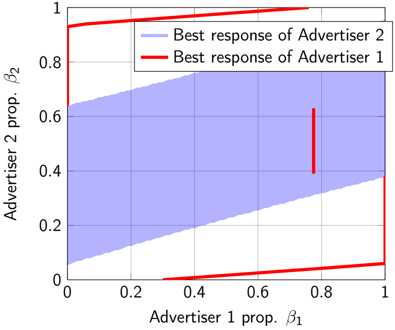
- To determine the Nash equilibria (if any), we define the *best response* of each advertiser as a function of the strategy of its opponent:

$$\begin{aligned} \mathrm{BR}_1(\beta_2) &= \arg\max_{\beta\in[0,1]} U_1(\beta, \beta_2) \text{ and} \\ \mathrm{BR}_2(\beta_1) &= \arg\max_{\beta\in[0,1]} U_2(\beta_1, \beta). \end{aligned}$$

- Graphically, the set of Nash equilibria is the (possibly empty) set of intersection points of BR curves.

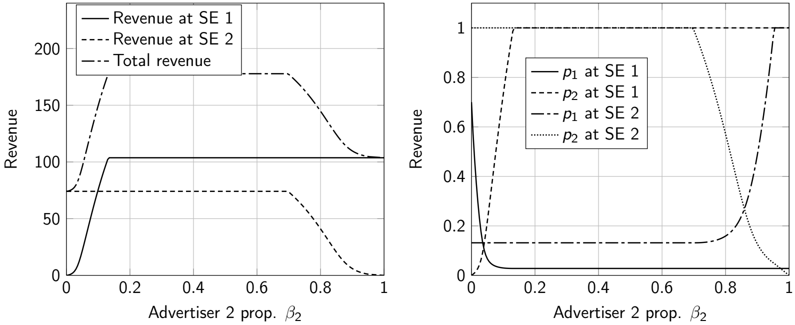
Illustration: both SEs implement bid-based pricing

$V \equiv U[0, 20]$, $\alpha = 0.6$, $\lambda = 100$, $q_1 = 0.5$, $q_2 = 0.6$, $b_1 = 5$, $c_1 = 0.5$, $b_2 = 20$, $c_2 = 0.4$, $v_1 = 10$, $v_2 = 9$ and $p_r = 0.1$



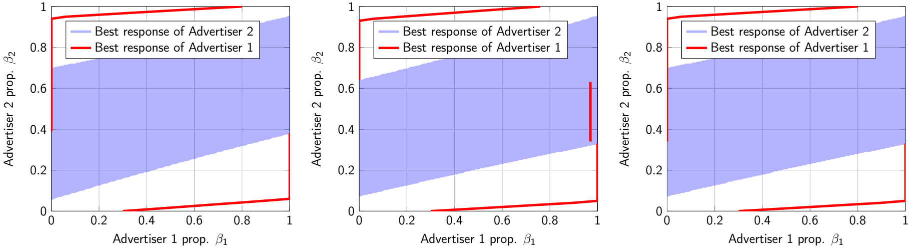
- for each fixed β_1, there is actually an interval for the best response of Advertiser 2 (blue)
- best response of Advertiser 1 in terms of β_2, we obtain the red curve
- set of Nash equilibria: $\{0.775\} \times [0.39, 0.63]$.

Explanation of an interval as best response, for $\beta_1 = 0.21$



- Left: revenue indeed constant
- Right: bidding probability constant
- The probability of bidding is maximal, equal to 1, independent of the submitted budget $\Rightarrow$ the budget is not fully spent.

3 other cases of ranking possibilities: bid based- revenue based (B-R), R-B and R-R



Nash equilibria:

- For the B-B case, all the profiles $(\beta_1, \beta_2) \in \{0.775\} \times [0.39, 0.63]$;
- for the B-R case, it is $\{0\} \times [0.39, 0.695]$;
- for the R-B case, $\{0.97\} \times [0.34, 0.63]$;
- for the R-R case, $\{0\} \times [0.34, 0.695]$.

Remark: when SE 2 implements revenue-based ranking, Advertiser 1's strategy at a Nash equilibrium is to put all her budget on SE 2.

Game between SEs on the ranking strategy

Anticipating the advertisers' decisions, the SEs seek to maximize their revenues from advertisement.

Another level of game: $(\text{Rev}_1, \text{Rev}_2)$ in terms of the rules used by SE 1 (line) and SE2 (column)

	B	R
B	$(10.15, 3.62)$	$(1.20, 11.06)$
R	$(11.32, 1.32)$	$(1.50, 11.06)$

- Best responses in red. Nash equilibrium: both elements in red.
- R-R: Nash equilibrium: in agreement with Yahoo!'s move to follow Google.
- For other sets of parameters such that B-R is an equilibrium. $\Rightarrow$ close look necessary for SEs!

Outline

Conclusions/Future activities

We have defined three models to investigate the impact of ranking strategies on advertisers behavior and search engines revenues

- shown that a random ranking can increase advertisers' benefits
- shown that search engines revenues are not always better with revenue-based ranking
- defined a model describing two search engines in competition, derived how advertisers should behave and illustrated how engines can (competitively) play on the ranking strategy.

Future activities:

- extend our study to the situation where SEs propose more than one slot
- look more closely at Google against Yahoo!
- look at the competitive case with our random ranking.