Games Among Telecommunication Network Providers

Bruno Tuffin (collaboration with Patrick Maillé)

Inria Rennes - Centre Bretagne Atlantique

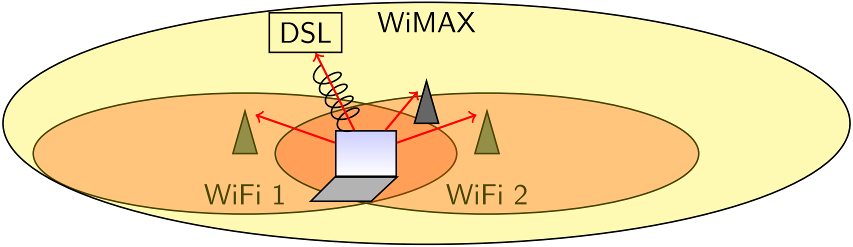
Evaluation Inria
Rungis/Orly, March 2012

Outline

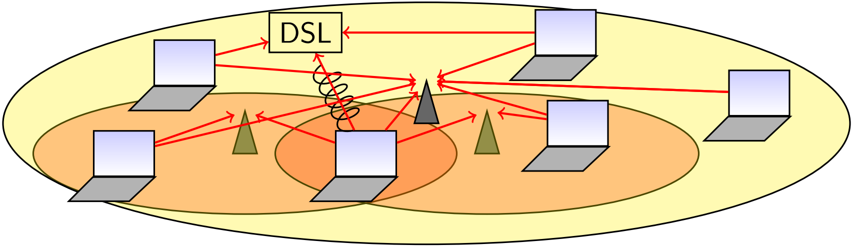
Outline

1. A general model
2. Competition on a common coverage area
3. WiFi against WiMAX
4. Other extensions: partial spectrum sharing & technological game

Specific model of competition among providers



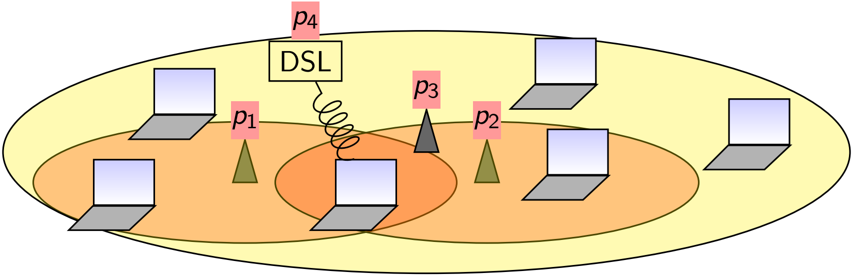
Specific model of competition among providers



- Interactions among non-cooperative consumers: *game*
- Congested networks provide poorer quality (packet losses)

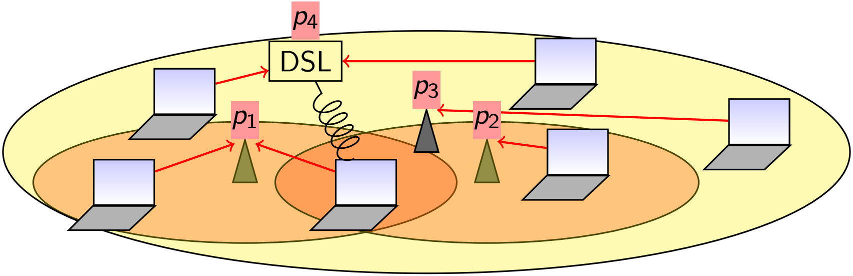
Specific model of competition among providers

But **providers** play first!



Specific model of competition among providers

But **providers** play first!

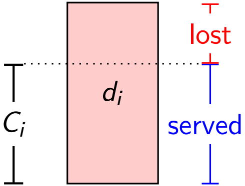


This work: study of the two-level noncooperative game.

1. *Higher level*: **providers** set prices to maximize revenue
2. *Lower level*: **consumers** choose their provider

Communication model: packet losses

- Time is slotted
- Each provider i has finite capacity C_i
- If total demand d_i at provider i exceeds C_i: exceeding packets are *randomly* lost



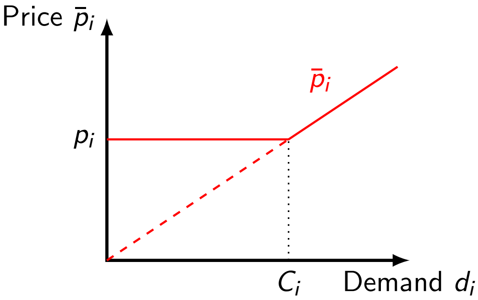
$$\mathbb{P}(\text{successful transmission}) = \min\left(1, \frac{C_i}{d_i}\right)$$

$$\Rightarrow \text{Expected number of transmissions} = \frac{1}{\mathbb{P}(\text{success})} = \max\left(1, \frac{d_i}{C_i}\right)$$

Only “regulation”: pay for what you send

The price p_i at each provider i is per packet *sent* Marbach'02
$\Rightarrow$ If several transmissions are needed, the user pays several times

$$\bar{p}_i := \textit{perceived} \text{ price at } i = \mathbb{E}[\text{price per packet}] = p_i \max\left(1, \frac{d_i}{C_i}\right)$$



Model for user choices: Wardrop equilibrium

- Users choose the provider(s) i with lowest $\bar{p}_i = p_i \max\left(1, \frac{d_i}{C_i}\right)$

$\Rightarrow$ For a given coverage zone Z, all providers with customers from that zone end up with the same perceived price $\bar{p}_i = \bar{p}_z$ Wardrop'52

Model for user choices: Wardrop equilibrium

- Users choose the provider(s) i with lowest $\bar{p}_i = p_i \max\left(1, \frac{d_i}{C_i}\right)$

$\Rightarrow$ For a given coverage zone Z, all providers with customers from that zone end up with the same perceived price $\bar{p}_i = \bar{p}_z$ Wardrop'52

- The total amount of data that users want to successfully transmit in a zone z depends on that price:

$$\sum_i d_{i,z} \min(1, C_i/d_i) = \alpha_z D(\bar{p}_z),$$

$$i.e. \quad \bar{p}_z = \underbrace{v}_{\text{marg. val. function}}\left(\frac{\sum_i d_{i,z} \min(1, C_i/d_i)}{\alpha_z}\right)$$

with D the total demand function, α_z the population proportion in zone z, and $d_{i,z}$ the demand in zone z for provider i.

Higher level: price competition game

- Providers set their price p_i *anticipating users reaction*
  $\Rightarrow$ Providers are Stackelberg leaders
- We can assume management costs of the form $\underbrace{\ell_i(d_i)}_{\text{nondecreasing, convex}}$

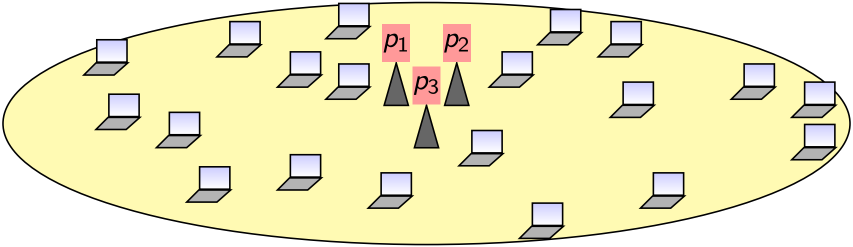
  Provider i's objective: $R_i := p_i d_i - \ell_i(d_i)$.

Outline

Competition model

Publications: Infocom'2008 & Jrnl sub.

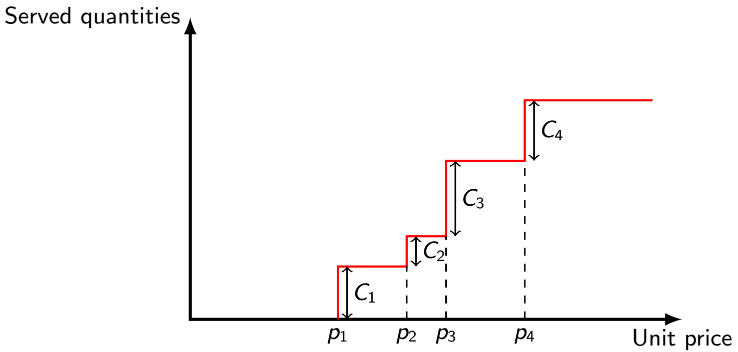
- Simplified topology: common coverage area
- N competing providers declaring price and capacity ($\mathcal{I} := \{1, \ldots, N\}$)



User equilibrium

- Users choose the provider(s) i with lowest $\bar{p}_i = p_i \max\left(1, \frac{d_i}{C_i}\right)$

$\Rightarrow$ All providers with customers end up with the same perceived price $\bar{p}_i = \bar{p}$ Wardrop'52



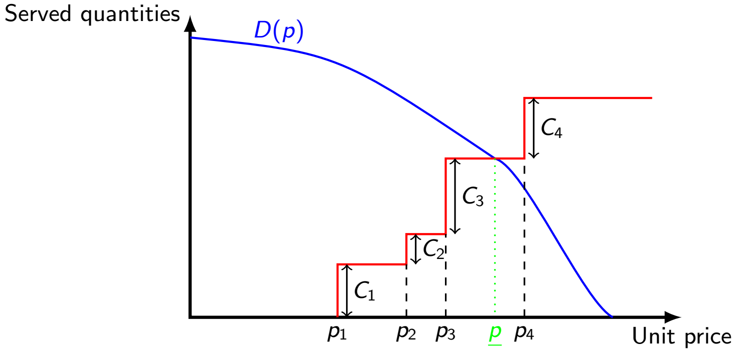
User equilibrium

- Users choose the provider(s) i with lowest $\bar{p}_i = p_i \max\left(1, \frac{d_i}{C_i}\right)$

$\Rightarrow$ All providers with customers end up with the same perceived price $\bar{p}_i = \bar{p}$ Wardrop'52

- The total demand level depends on that price:

$$\bar{p} = \underbrace{v}_{\text{marg. val. function}}\left(\sum \min(C_i, d_i)\right)$$



Price competition, main result

Proposition

Under sufficient condition A, there exists a **unique Nash equilibrium** *on price war among providers, given by*

$$\forall i \in \mathcal{I}, \quad \begin{cases} p_i &= v\left(\sum_{j\in\mathcal{I}} C_j\right) \\ d_i &= C_i. \end{cases}$$

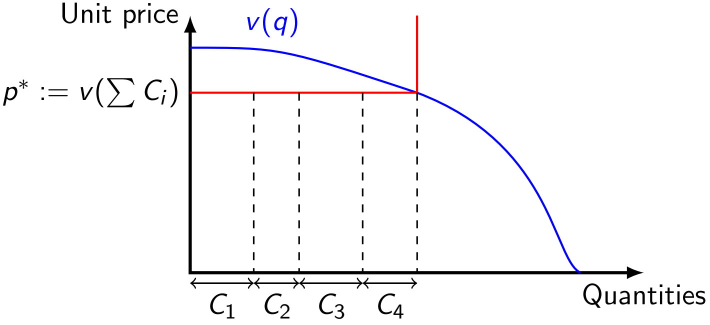
- **Sufficient condition A:** each ℓ_i is Lipschitz with constant κ_i, and $\forall y \geq p^* := v\left(\sum_{j\in\mathcal{I}} C_j\right)$, the demand function D is sufficiently elastic:
$$\frac{-yD'(y)}{D(y)} \geq \frac{1}{1-\kappa/y}, \tag{1}$$
where $\kappa := \max_{i\in\mathcal{I}} \kappa_i$.
- Without cost functions, it just means a demand elasticity larger than -1.

Price competition, main result

Proposition

Under sufficient condition A, there exists a **unique Nash equilibrium** *on price war among providers, given by*

$$\forall i \in \mathcal{I}, \quad \begin{cases} p_i &= v\left(\sum_{j\in\mathcal{I}} C_j\right) \\ d_i &= C_i. \end{cases}$$



Social Welfare considerations

- A performance measure of the outcome $(d_1, ..., d_I)$ of the game $=$ overall value of the system

$$\text{Social Welfare} := \int_{u=0}^{\sum_{i\in\mathcal{I}} d_i} \frac{\sum_{i\in\mathcal{I}} \min(d_i, C_i)}{\sum_{i\in\mathcal{I}} d_i} v(u)du - \sum_i \ell_i(d_i).$$

- **Remark:** the Social Welfare maximization problem leads to the same outcome $d_i = C_i \quad \forall i$ as the price war.
- **Consequence:** The Nash equilibrium corresponds to the socially optimal situation: the Price of Anarchy is 1!.
- Additional result:

Proposition

Under the same conditions about **demand elasticity***, no provider can increase its revenue by artificially lowering its capacity.*

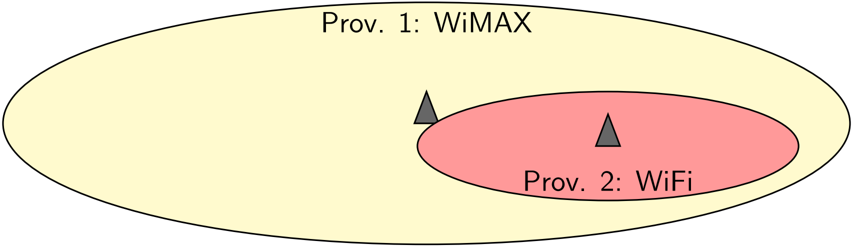
Outline

Competition model

Publications: Computer Networks 2010, ICQT'09

Assumptions

- Two competing providers declaring price and capacity
- One coverage area included in the other

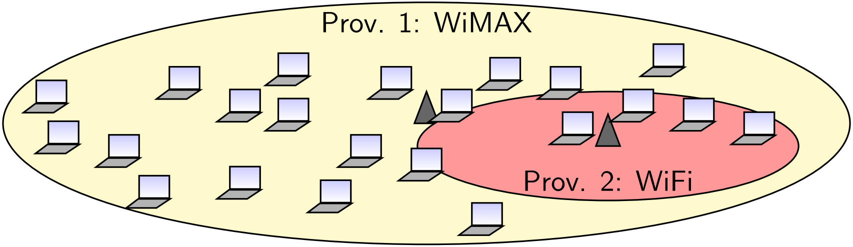


Competition model

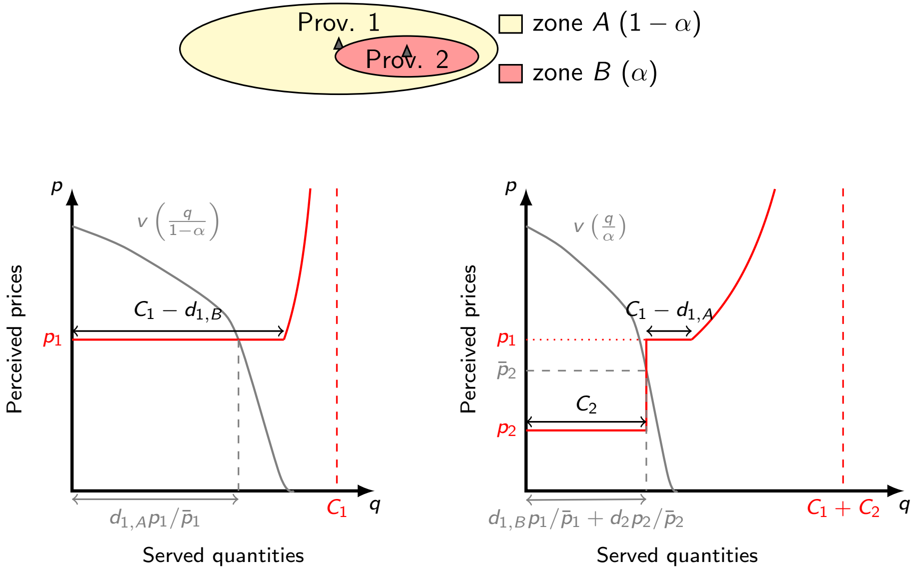
Publications: Computer Networks 2010, ICQT'09

Assumptions

- Two competing providers declaring price and capacity
- One coverage area included in the other



User equilibrium: illustration



User equilibrium: existence and uniqueness

Proposition

For all price profile, there exists at least a user (Wardrop) equilibrium. Moreover, the corresponding perceived prices of each provider are unique.

NB: demand repartition among providers is not necessarily unique.

Higher level: price competition game

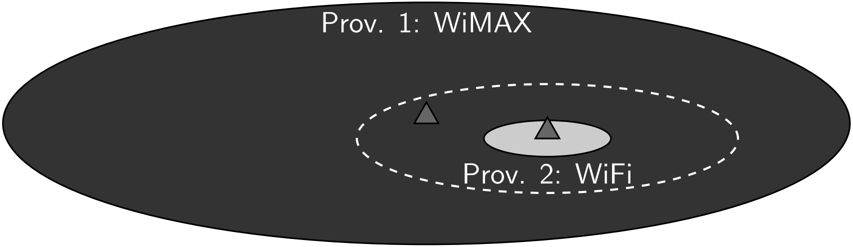
- Provider i's objective: $R_i := p_i d_i - \ell_i(d_i)$.

Proposition

If $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* (p_1^*, p_2^*) *in the price war between providers.*

- *If* $\alpha \leq \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

(Darker=more expensive)

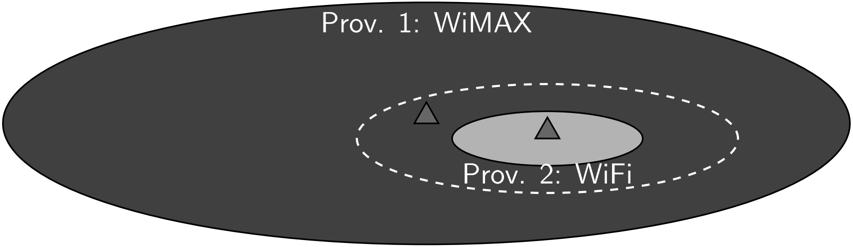


Proposition

If $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* (p_1^*, p_2^*) *in the price war between providers.*

- *If* $\alpha \leq \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

(Darker=more expensive)

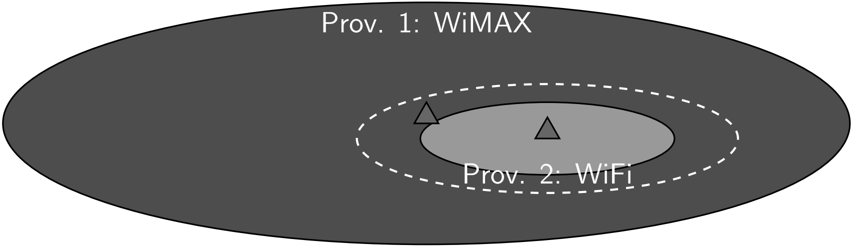


Proposition

If $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* (p_1^*, p_2^*) *in the price war between providers.*

- *If* $\alpha \leq \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

(Darker=more expensive)

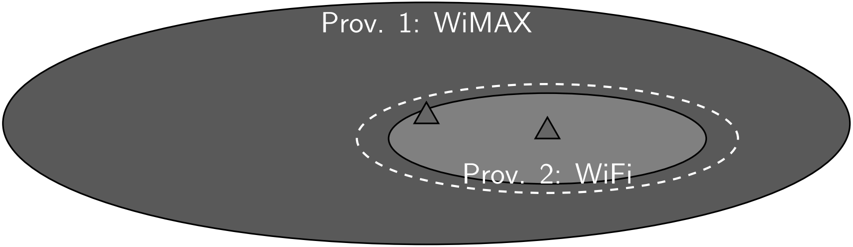


Proposition

If $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* (p_1^*, p_2^*) *in the price war between providers.*

- *If* $\alpha \leq \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

(Darker=more expensive)

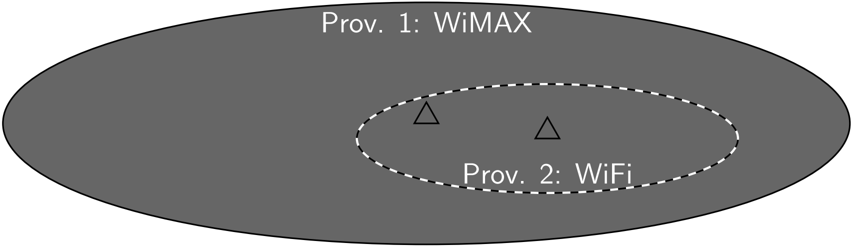


Proposition

If $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* (p_1^*, p_2^*) *in the price war between providers.*

- *If* $\alpha \leq \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

(Darker=more expensive)

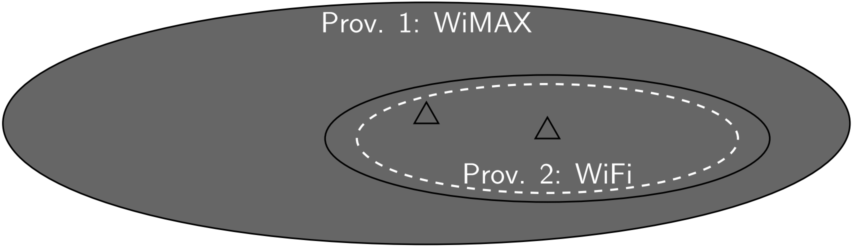


Proposition

If $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* (p_1^*, p_2^*) *in the price war between providers.*

- *If* $\alpha \leq \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

(Darker=more expensive)

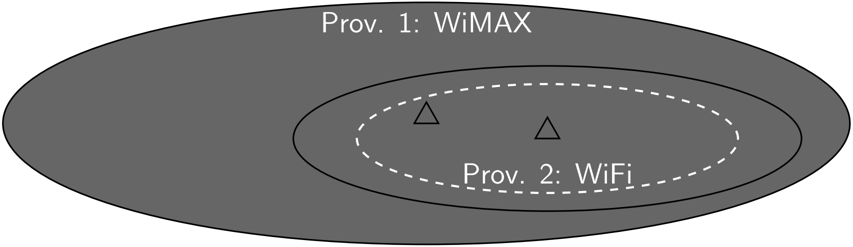


Proposition

If $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* (p_1^*, p_2^*) *in the price war between providers.*

- *If* $\alpha \le \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \ge p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

(Darker=more expensive)

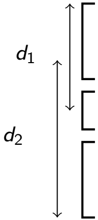


Outline

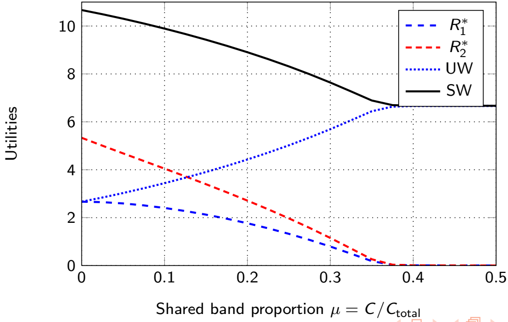
Partial spectrum sharing

Publication: Globecom'09

Each provider i still has some "private" band C_i, but **an amount C of spectrum has to be shared among providers**



$$C = \begin{cases} C_1' = \dfrac{[d_1 - C_1]^+}{[d_1-C_1]^+ + [d_2-C_2]^+} C \\ C_2' = \dfrac{[d_2 - C_2]^+}{[d_1-C_1]^+ + [d_2-C_2]^+} C \end{cases}$$



Technological Investment Games Publications: IJNM'11, Globecom'10

- Should a provider invest in infrastructure or/and licence? I.e.,
  - invest on new technologies (WiMAX, new 3G license...)
  - maintain existing ones (WiFi, 3G...).
- Three-level games for three time scales:
  - Lowest level game: Wardrop equilibrium for users
    - Users (infinitesimal) have terminals with multiple interfaces and choose the "best" couple (provider, technology) depending on QoS and prices
    - There always exists a user equilibrium.
  - Intermediate level: pricing game
    - For any fixed set of implemented technologies per provider
    - Game knowing what would be the user equilibrium
    - Determination of a Nash equilibrium (if any).
  - Highest level: Technological game
    - Providers choose their subset $\mathcal{S}_i$ of implemented technologies, resulting in a (multidimensional) matrix of revenues $(R_1(\mathcal{S}), \ldots, R_N(\mathcal{S}))_{\mathcal{S}}$ with $\mathcal{S} = (\mathcal{S}_i)_i$. from the above game
    - and a cost matrix $C = (c_1(\mathcal{S}), c_2(\mathcal{S}))_{\mathcal{S}_1, \mathcal{S}_2 \subset \mathcal{T}}$.
    - Goal of each provider i: maximize net benefit

$$B_i(\mathcal{S}) = R_i(\mathcal{S}) - \sum_{t \in \mathcal{S}_i} c_{i,t} = \sum_{t \in \mathcal{S}_i} (p_i^* d_{i,t}^* - c_{i,t}).$$

A case study as an illustration: a WiFi-positionned provider against a 3G one

- a WiFi-installed provider (1), Free, wishing to extend her position against a 3G-installed provider (2), Orange.
- Cost of the fourth licence in France (Free is buying): 240 M€.

1 \2	∅	3G	WiM.	3G,WiM.	WiFi	WiFi,3G	WiFi,WiM.	WiFi,3G,WiM.
∅	0;0	0;1929	0;2555	0;3716	0;2178	0;3629	0;4047	0;**4778**
3G	1437;0	1167;1679	1057;2198	810;3141	1208;1935	937;3161	826;3493	590;**4000**
WiMAX	2555;0	2198;1549	2040;2040	1665;2875	2237;1837	1865;2954	1708;3238	**1368**;**3628**
3G,WiMAX	3224;0	2649;1302	2383;1665	1781;2273	**2715**;1616	**2100**;2488	**1834**;2664	1235;**2817**
WiFi	2228;0	1985;1700	1887;2237	1666;**3207**	0;-50	-	-	-
WiFi,3G	3187;0	2719;1429	2512;1865	2046;**2592**	-	-	-	-
WiFi,WiM.	4097;0	3543;1318	**3288**;1708	**2714**;**2326**	-	-	-	-
WiFi,3G,WiM.	**4336**;0	**3558**;1082	3186;1368	2375;**1727**	-	-	-	-

- Two non-symetric Nash equilibria. No investment on 3G for Free
- By reducing a bit more the licence cost, 3G investment for Free: threshold easy to compute.