Symbolic Unfolding Of Time Petri Nets

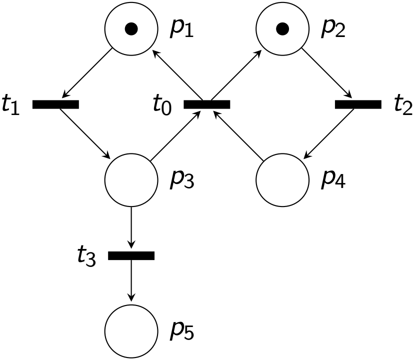
Collaboration INRIA/Distribcom & CNRS/IRCCyN

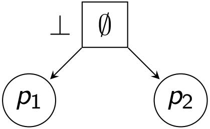
March 22, 2012

Introduction

- Objectives:
  - Modeling and analysis of **concurrent timed** systems,
  - Explicit the causality between events and **preserve** concurrency,
  - Infer constraints on the value of **parameters**.
- Applications in the Distribcom group:
  - Diagnosis (root cause analysis),
  - Causality-based testing,
  - QoS evaluation,
  - Concurrent semantics of timed languages (e.g. ORC).
- Main contribution:
  - Unfoldings of TPNs are not TPNs: new idea of symbolic unfoldings and conditions for having finite prefixes (a ten years old problem).

Petri nets and branching processes





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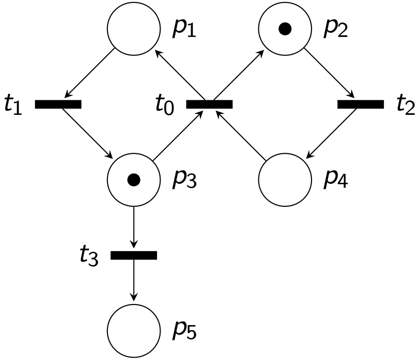

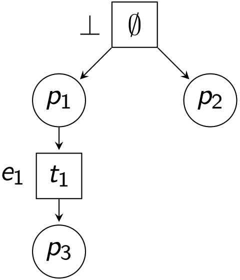

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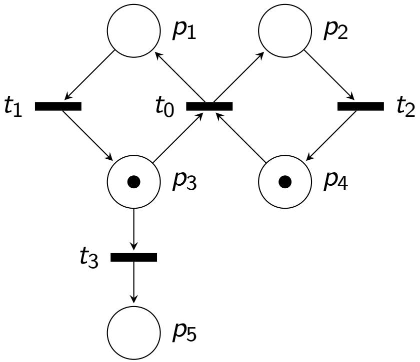

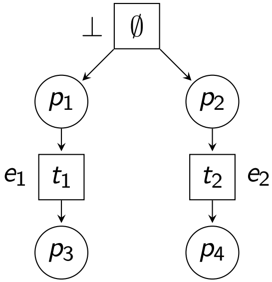

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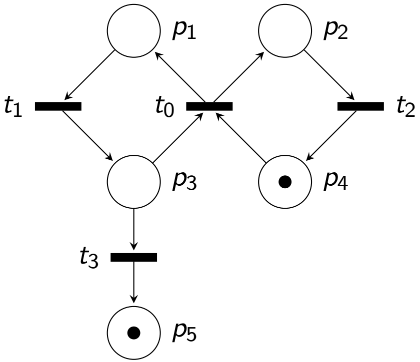

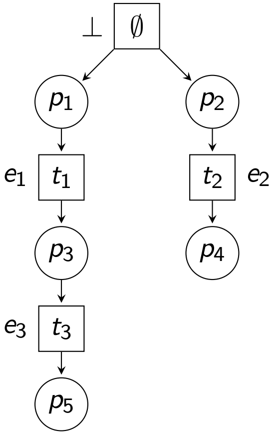

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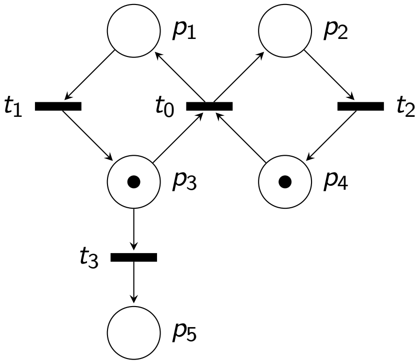

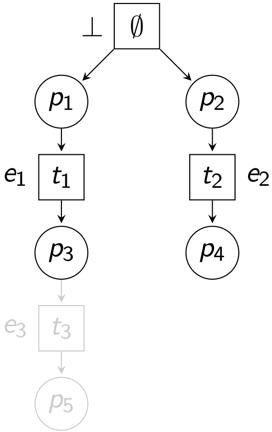

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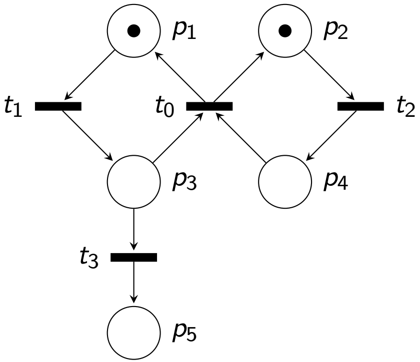

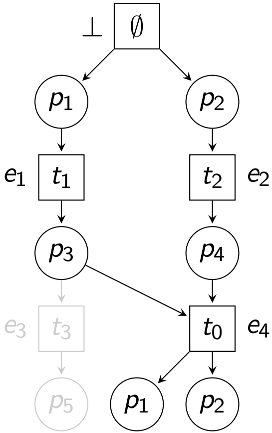

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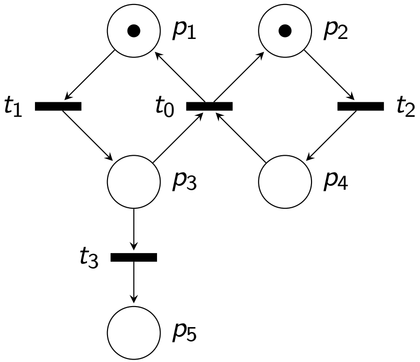
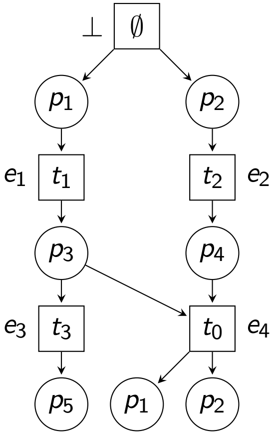
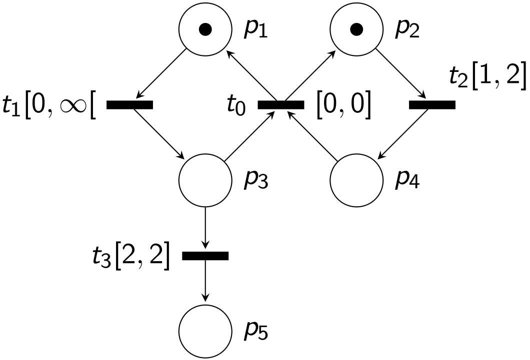
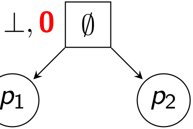




The **unfolding** of the Petri net is the **greatest** branching process.

Time Petri nets (TPN) and time processes





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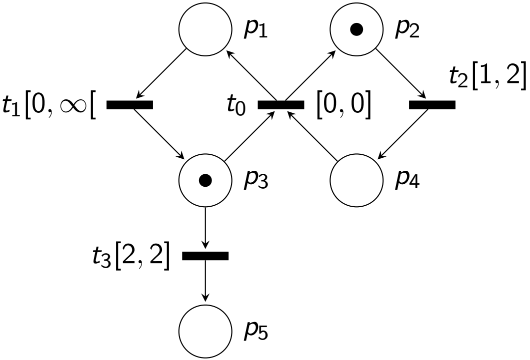

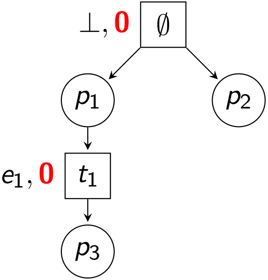

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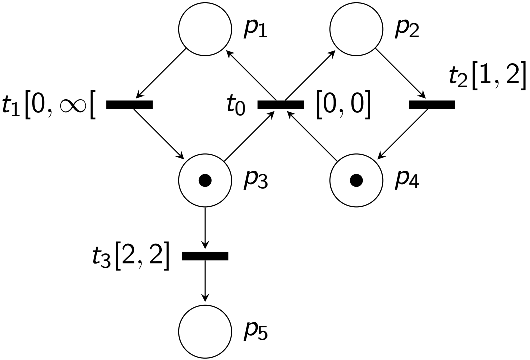

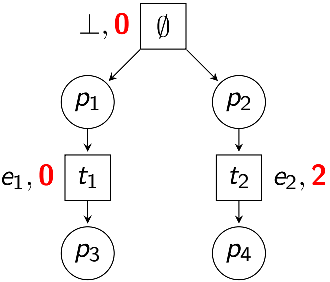

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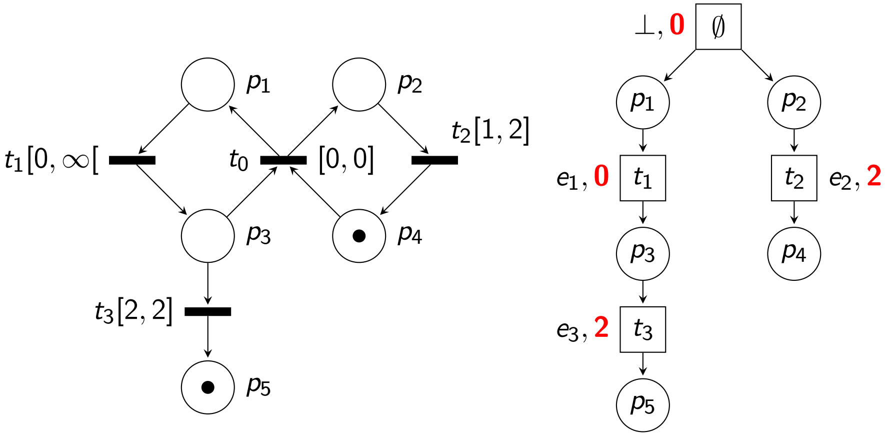

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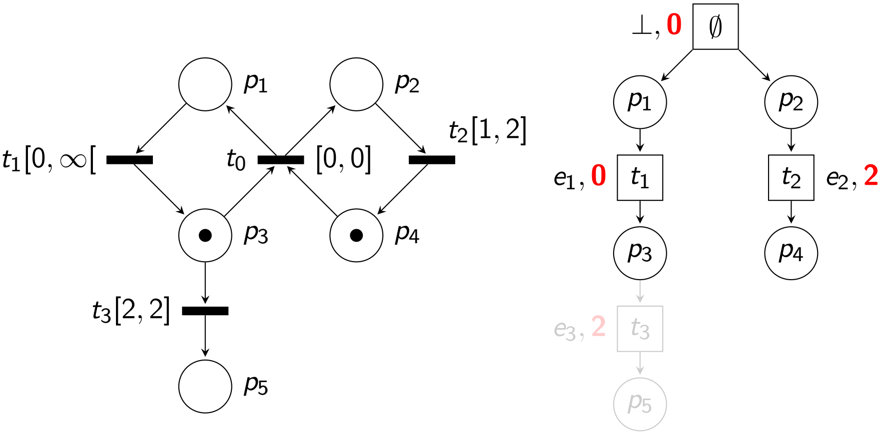

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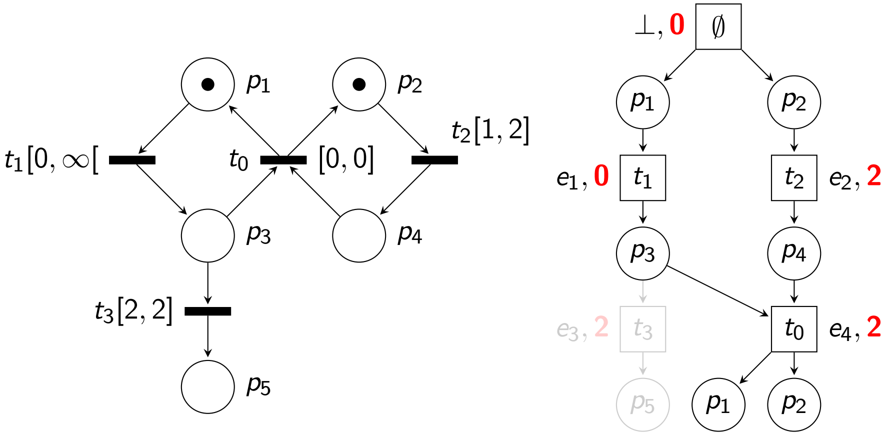

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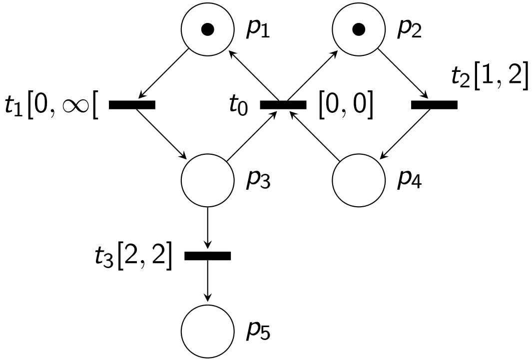
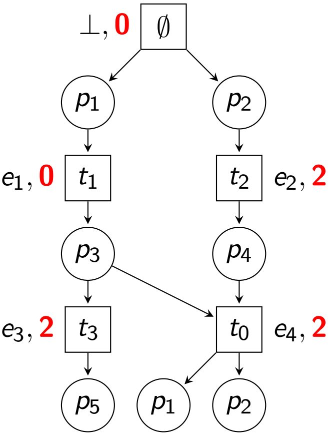
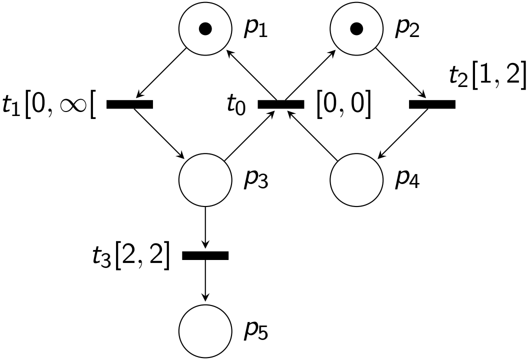
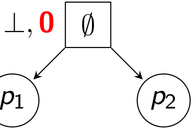




From time processes to time branching processes





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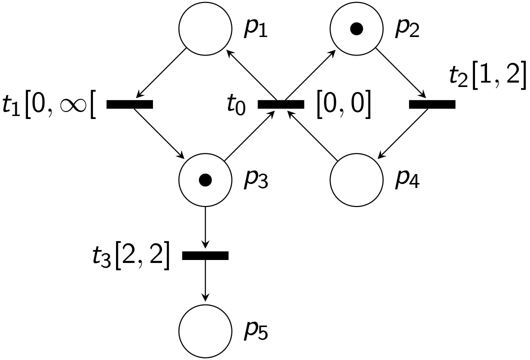

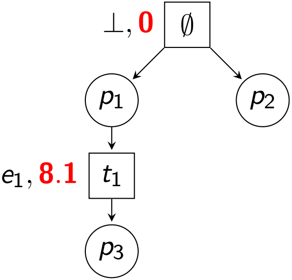

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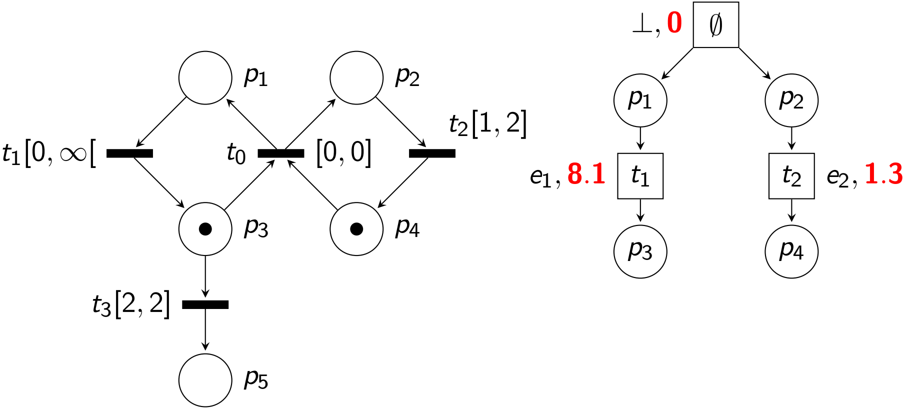

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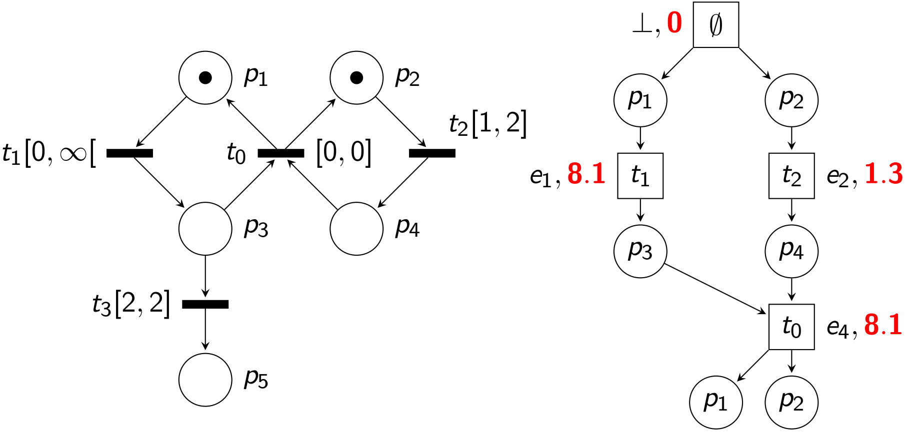

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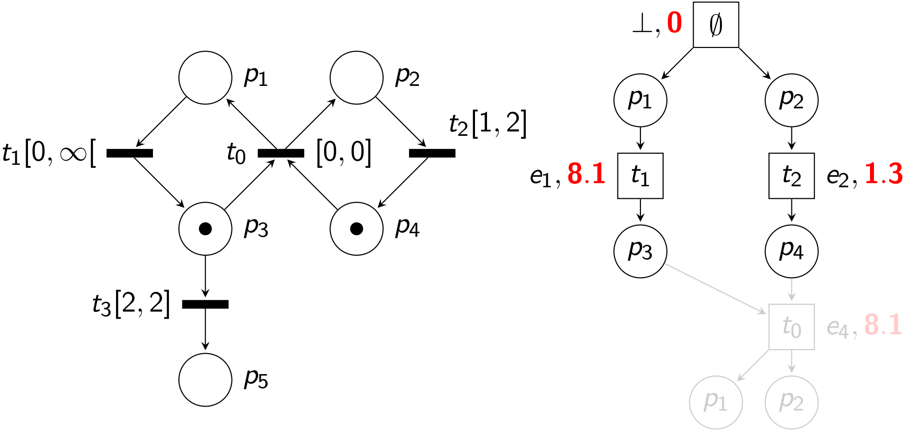

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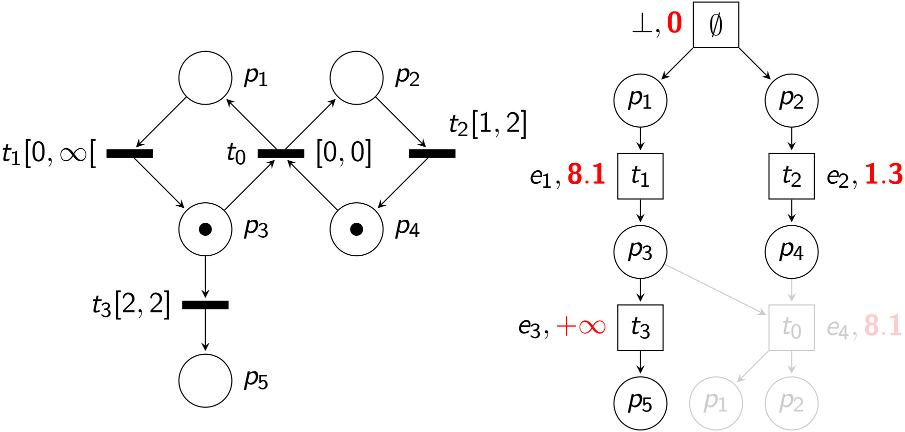

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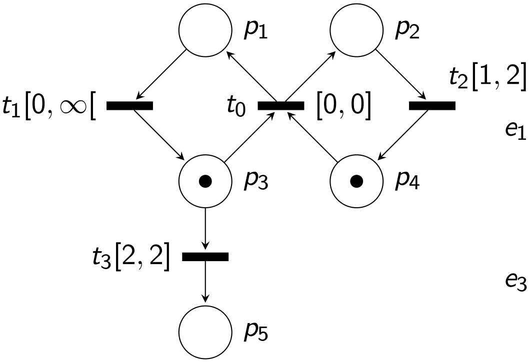
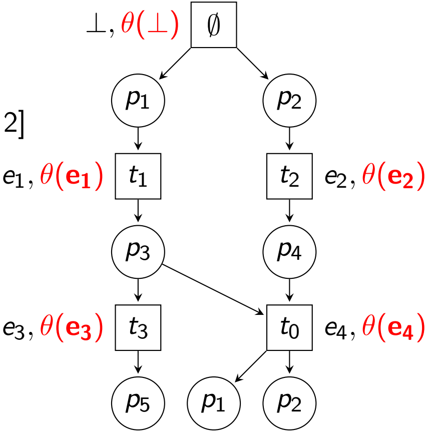
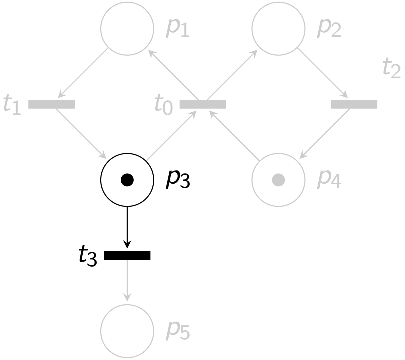


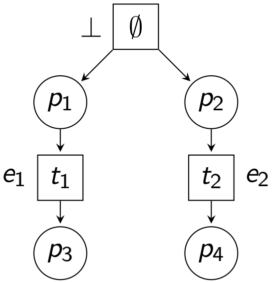


The **symbolic unfolding** of the time Petri net is the **union** of all time branching processes.

$$\left\{ \begin{array}{l} \theta(\bot) = 0, \\ \theta(e_1) = 0, \\ \theta(e_2) = 2, \\ \theta(e_3) = 2, \\ \theta(e_4) = +\infty \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} \theta(\bot) = 0, \\ 0 \leq \theta(e_1), \\ 1 \leq \theta(e_2) \leq 2, \\ \theta(e_3) = +\infty, \\ \theta(e_4) = \max(\theta(e_1), \theta(e_2)) \end{array} \right.$$

Locality





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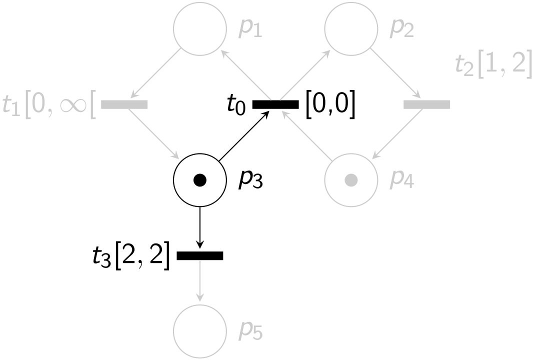

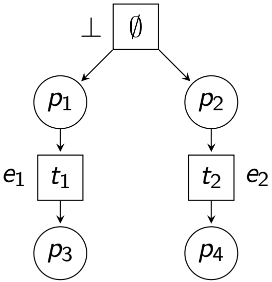

Conflicts with **urgency** break the locality of the firing rule.

Valid Time Branching Processes

We define the following constraints for a time labeling θ to be **valid**:

The transition fires in its interval

$$\Big[\theta(e) \neq \infty \ \wedge \ \theta(e) - \mathrm{TOE}(e) \in \big[\mathrm{eft}(l(e)), \mathrm{lft}(l(e))\big]$$

where ${}^{\bullet}e$ is the set of preconditions of e, $l(e)$ is the transition labeling e,
$\mathrm{TOE}(e) = \max(\theta(b) | b \in {}^{\bullet}e)$.

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. . . and no transition in direct conflict fires

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...one of its precondition was never produced

$$\vee \ \Big[\theta(e) = \infty \ \wedge \ \exists b \in {}^{\bullet}e, \ \theta({}^{\bullet}b) = \infty\Big]$$

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. . . a directly conflicting transition fires before

$$\vee \ \Big[\theta(e) = \infty \ \wedge \ \exists e' \text{ conf } e \wedge \ \theta(e') \neq \infty \wedge \theta(e') - \mathsf{TOE}(e) \leq \mathsf{lft}(l(e))\Big]$$

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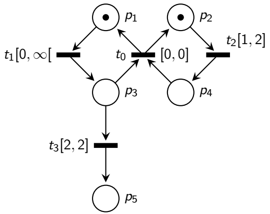
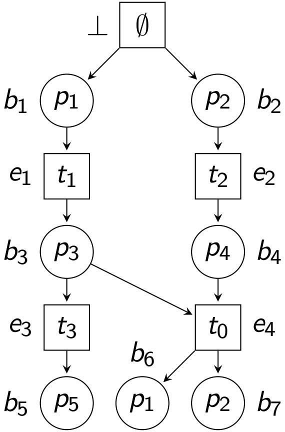
Infinite unfoldings and finite prefixes





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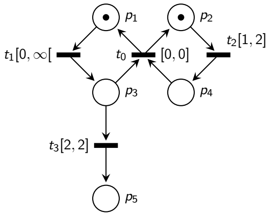

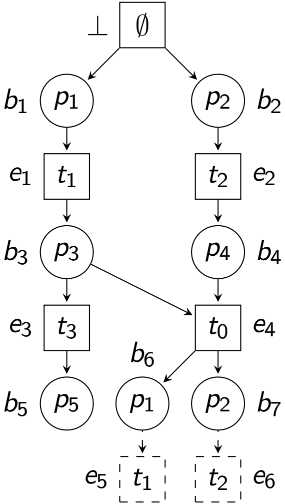

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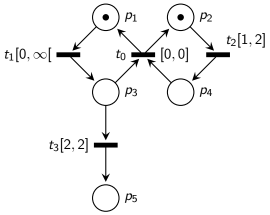
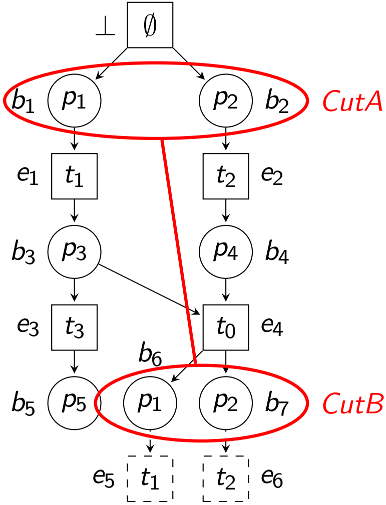


- $l(CutA) = l(CutB) = \{p_1, p_2\}$



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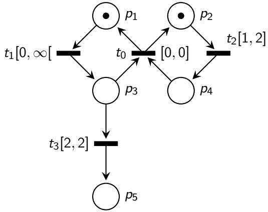

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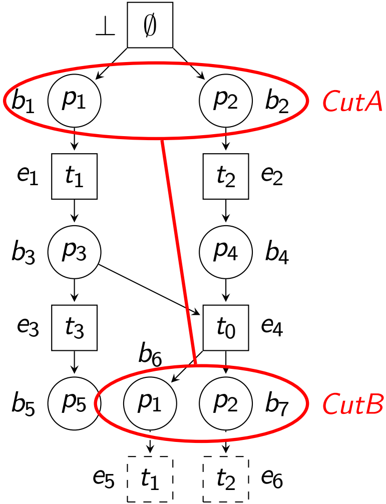

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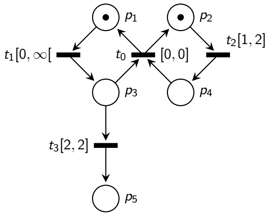
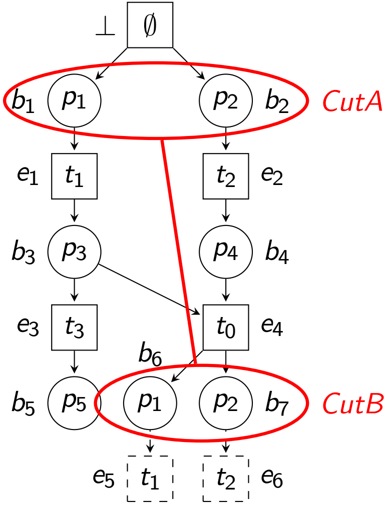


- $l(CutA) = l(CutB) = \{p_1, p_2\}$
- $l(b_1) = l(b_6) = p_1$
- age$(b_1) =$ $\min\{5, \max\{\theta(^\bullet b_1), \theta(^\bullet b_2)\} - \theta(^\bullet b_1)\} = 0$



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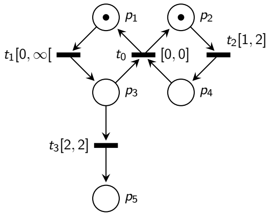

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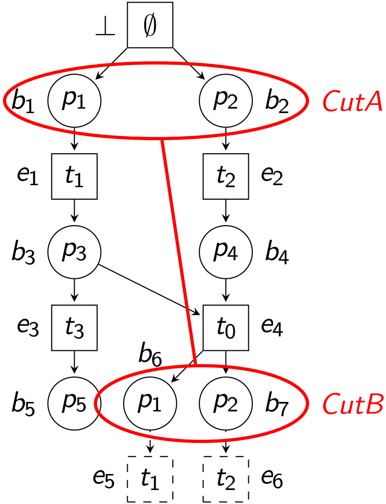
- $\mathrm{age}(b_1) = \min\{5, \max\{\theta({}^\bullet b_1), \theta({}^\bullet b_2)\} - \theta({}^\bullet b_1)\} = 0$
- $\mathrm{age}(b_6) = \min\{5, \max\{\theta({}^\bullet b_6), \theta({}^\bullet b_7)\} - \theta({}^\bullet b_6)\} = 0 = \mathrm{age}(b_1)$



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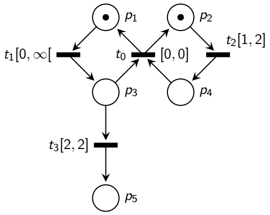

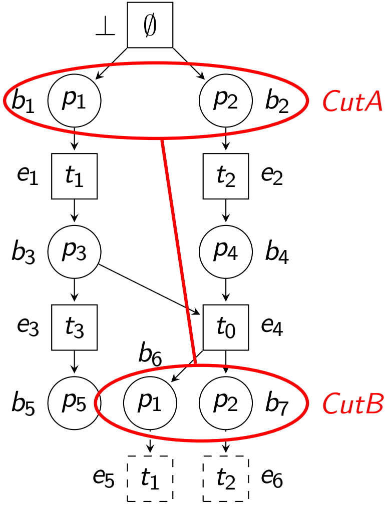

- $l(CutA) = l(CutB) = \{p_1, p_2\}$
- $l(b_1) = l(b_6) = p_1$
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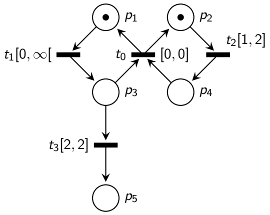
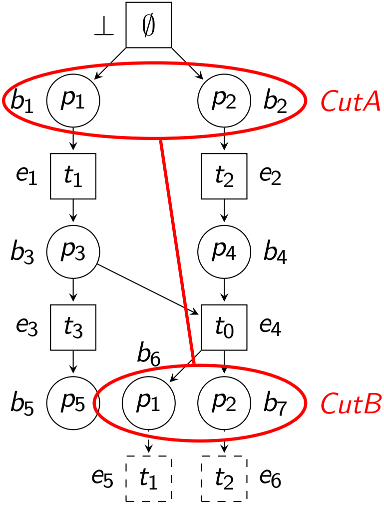
- $\text{age}(b_6) = \min\{5, \max\{\theta(^\bullet b_6), \theta(^\bullet b_7)\} - \theta(^\bullet b_6)\} = 0 = \text{age}(b_1)$
- Similarly, $l(b_2) = l(b_7) = p_2$ and $\text{age}(b_2) = \text{age}(b_7) = 0$

Infinite unfoldings and finite prefixes





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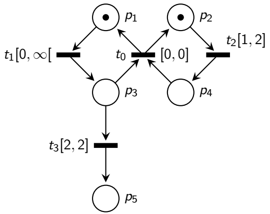
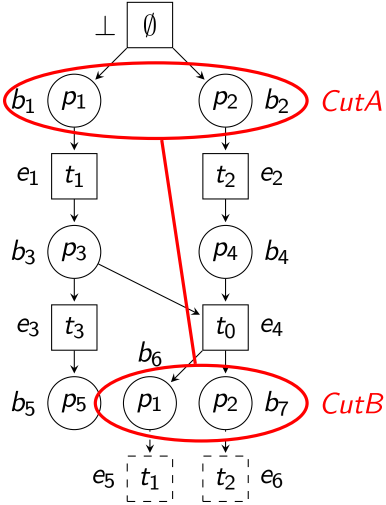
- $\mathrm{age}(b_6) = \min\{5, \max\{\theta({}^\bullet b_6), \theta({}^\bullet b_7)\} - \theta({}^\bullet b_6)\} = 0 = \mathrm{age}(b_1)$
- Similarly, $l(b_2) = l(b_7) = p_2$ and $\mathrm{age}(b_2) = \mathrm{age}(b_7) = 0$
- *CutA* and *CutB* are **equivalent**

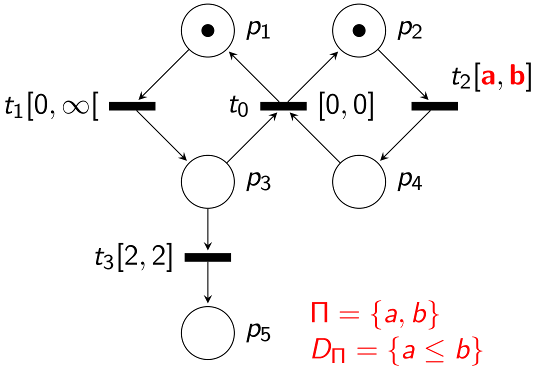
Infinite unfoldings and finite prefixes



- $I(CutA) = I(CutB) = \{p_1, p_2\}$
- $I(b_1) = I(b_6) = p_1$
- $\text{age}(b_1) =$ $\min\{5, \max\{\theta(^\bullet b_1), \theta(^\bullet b_2)\} - \theta(^\bullet b_1)\} = 0$
- $\text{age}(b_6) = \min\{5, \max\{\theta(^\bullet b_6), \theta(^\bullet b_7)\} - \theta(^\bullet b_6)\} = 0 = \text{age}(b_1)$
- Similarly, $I(b_2) = I(b_7) = p_2$ and $\text{age}(b_2) = \text{age}(b_7) = 0$
- $CutA$ and $CutB$ are **equivalent**
- e_5 and e_6 are **cutoff** events

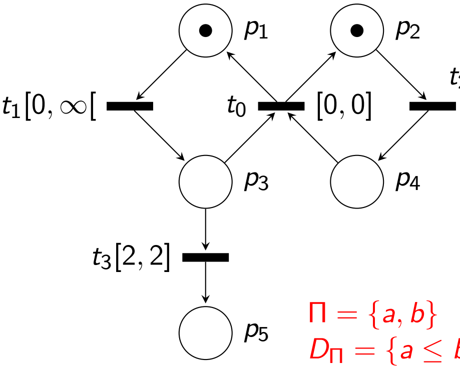


Parametric TPNs (PTPNs)

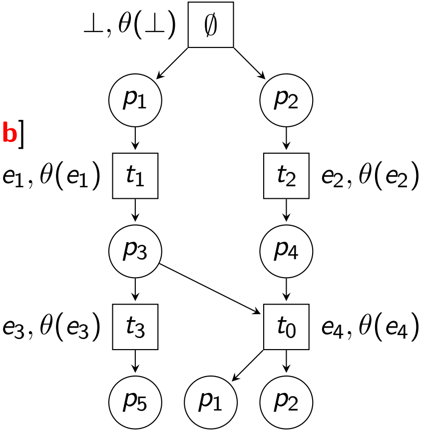


$\Pi = \{a, b\}$
$D_\Pi = \{a \leq b\}$

Parametric TPNs (PTPNs)



$\Pi = \{a, b\}$
$D_\Pi = \{a \leq b\}$



The **symbolic unfolding** of the PTPN is the **union** of all time branching processes for all feasible **parameter valuations**.

$$\begin{cases} \theta(\perp) = 0, \\ 0 \leq \theta(e_1), \\ \mathbf{a} \leq \theta(e_2) \leq \mathbf{b}, \\ \theta(e_3) = \theta(e_1) + 2, \\ \theta(e_3) \leq \theta(e_2) \\ \theta(e_4) = +\infty \end{cases} \quad \text{or} \quad \begin{cases} \theta(\perp) = 0, \\ 0 \leq \theta(e_1), \\ \mathbf{a} \leq \theta(e_2) \leq \mathbf{b}, \\ \theta(e_3) = +\infty, \\ \theta(e_4) = \max(\theta(e_1), \theta(e_2)) \end{cases}$$

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