Networks, Graphs and Algorithms

GANG (of Networks)

INRIA Paris Rocq. – LIAFA (Paris Diderot Univ., CNRS)

Laurent Viennot

Team Members

INRIA

- Laurent Viennot (DR)
- Dominique Fortin (CR)
- Fabien Mathieu (R)

CNRS

- Pierre Fraigniaud (DR)
- Amos Korman (CR)

Paris Diderot University

- Yacine Boufkhad (MdC)
- Pierre Charbit (MdC)
- Fabien de Montgolfier (MdC)
- Carole Delporte (Prof)
- Hugues Fauconnier (MdC)
- Michel Habib (Prof)

PhD Students

- Heger Arfaoui
- Hervé Baumann
- Jérémie Dusart
- Xavier Koegler
- Antoine Mamcarz
- Hung Tran-The

Administrative assistant

- Christine Anocq

Goals

- Theory of networks
- Algorithms for networks

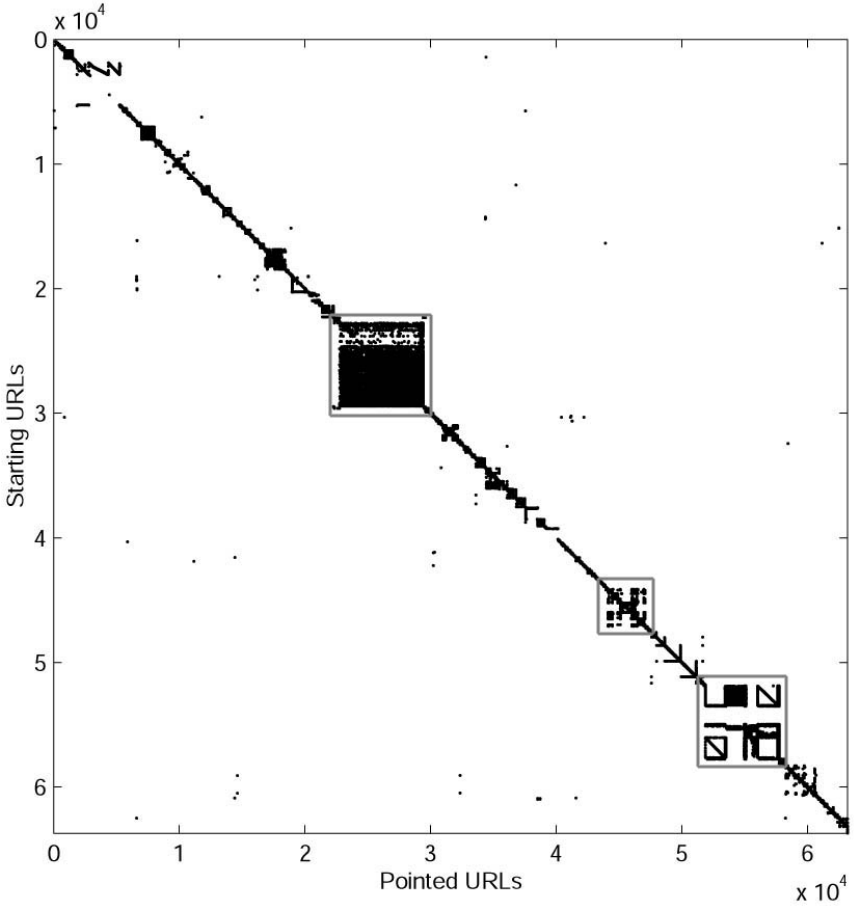
Challenges

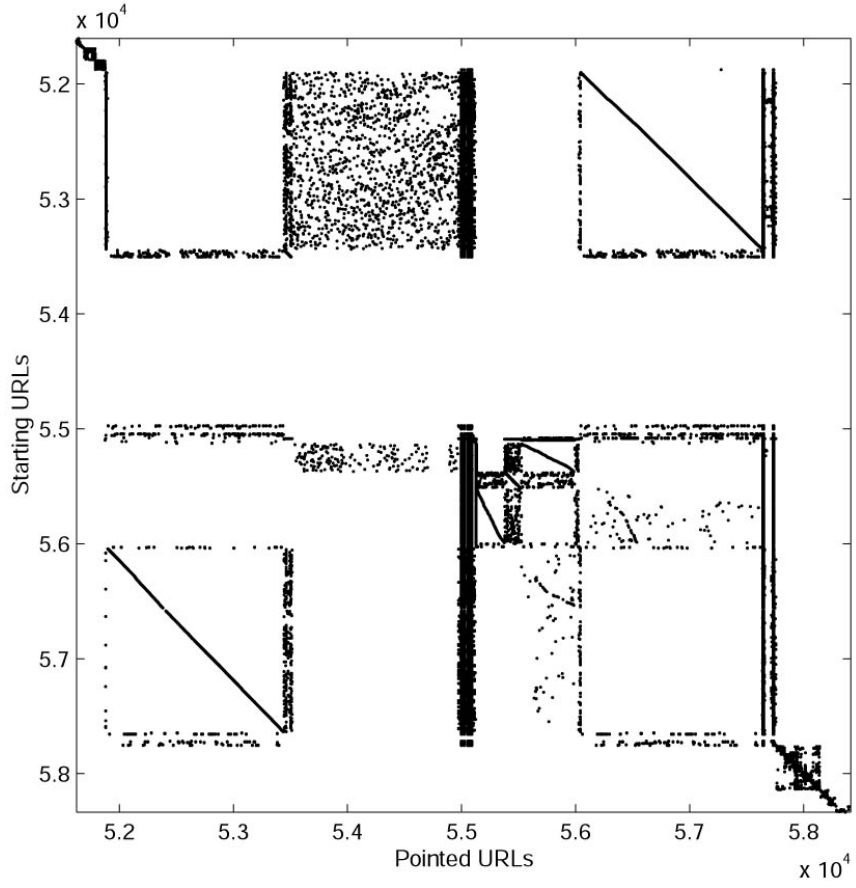
- Networks get larger and larger
- Manage large scale mobility : fusion of Internet and radio networks
- Massively distributed applications : between cloud and P2P

Approach

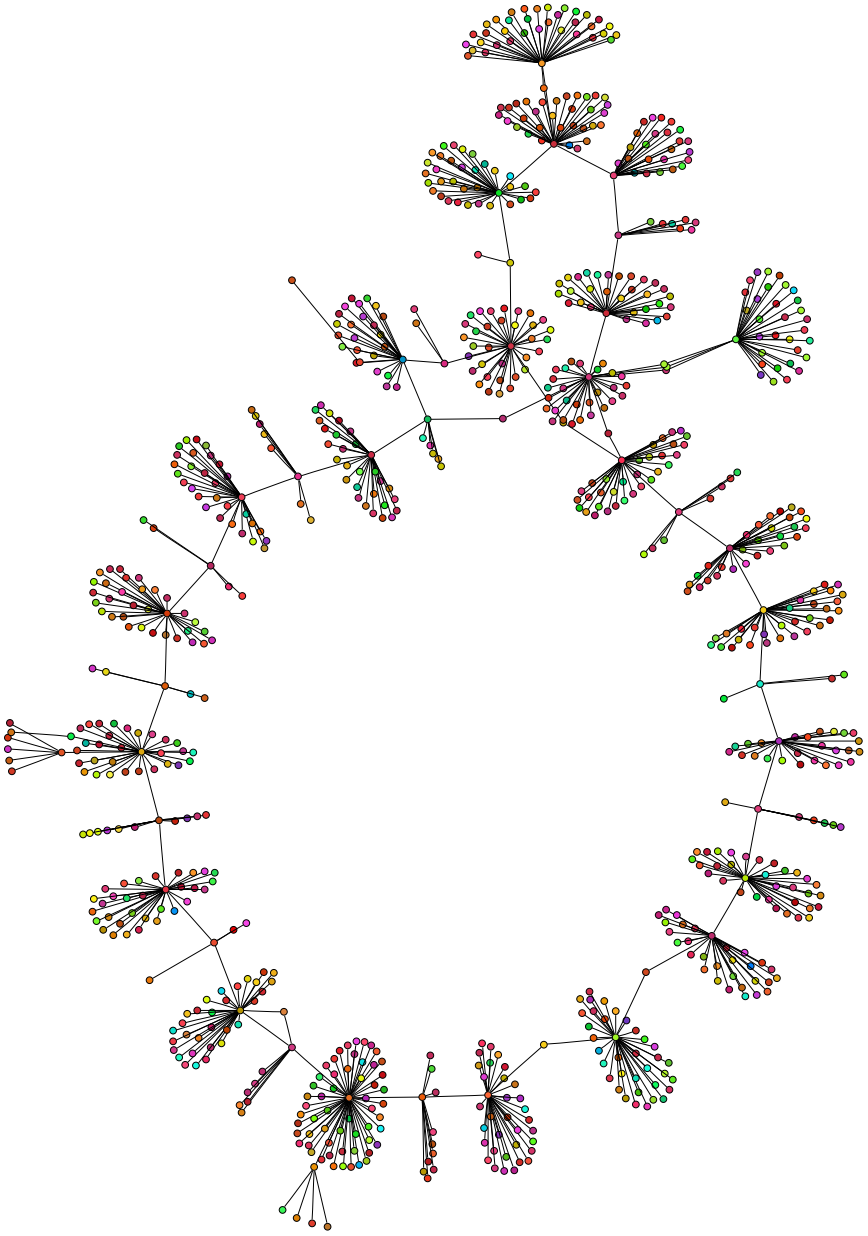
- Identify inherent structures in networks or applications to derive efficient algorithms.
- Bring powerful theoretical tools toward practice.

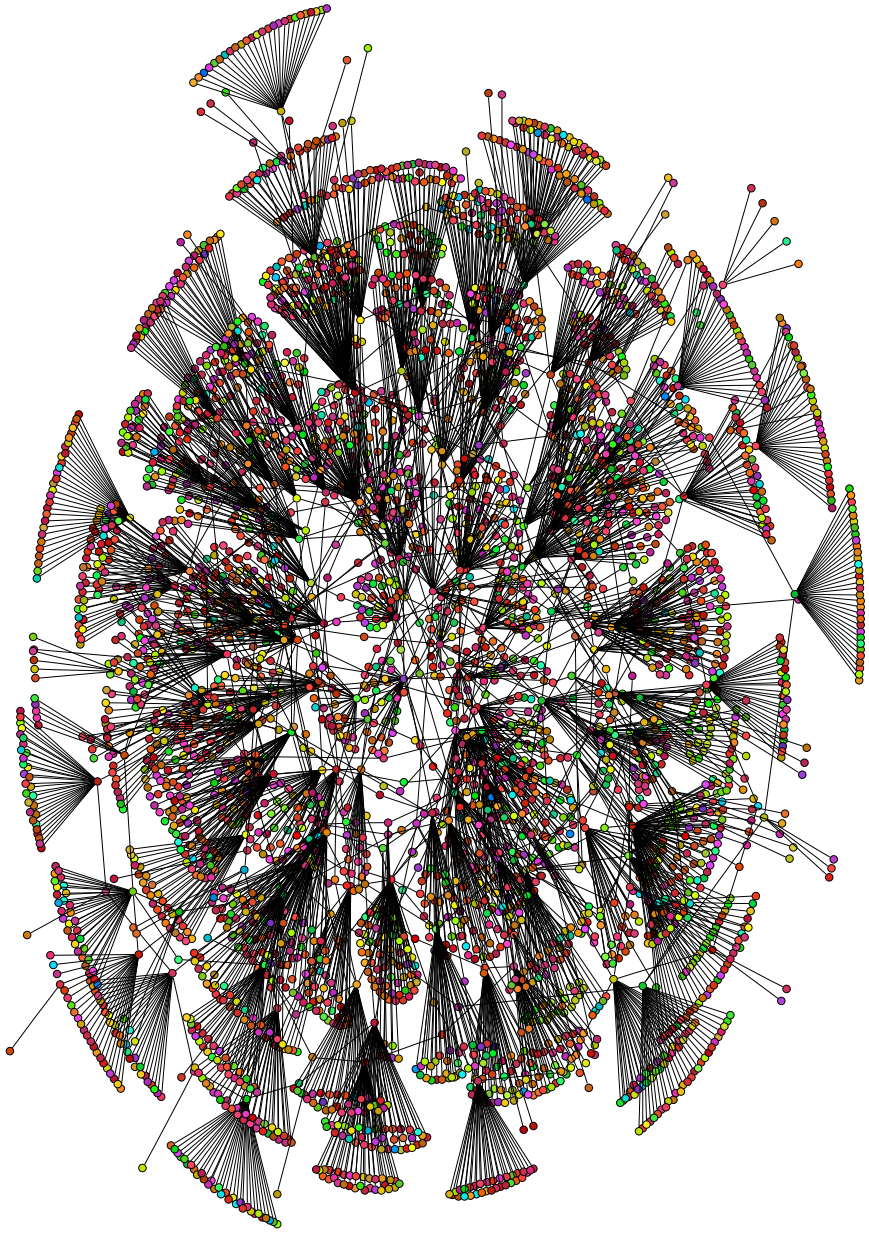
Web Adjacency Matrix



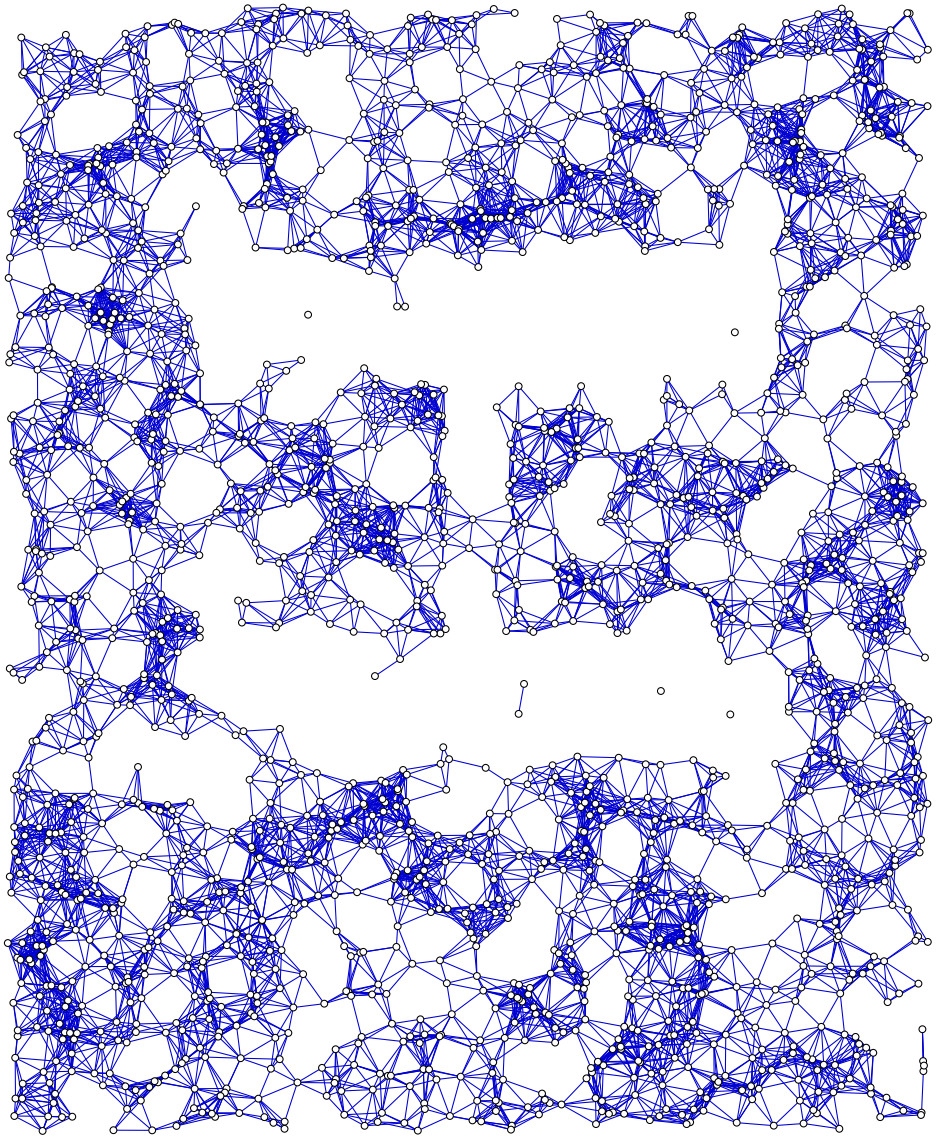


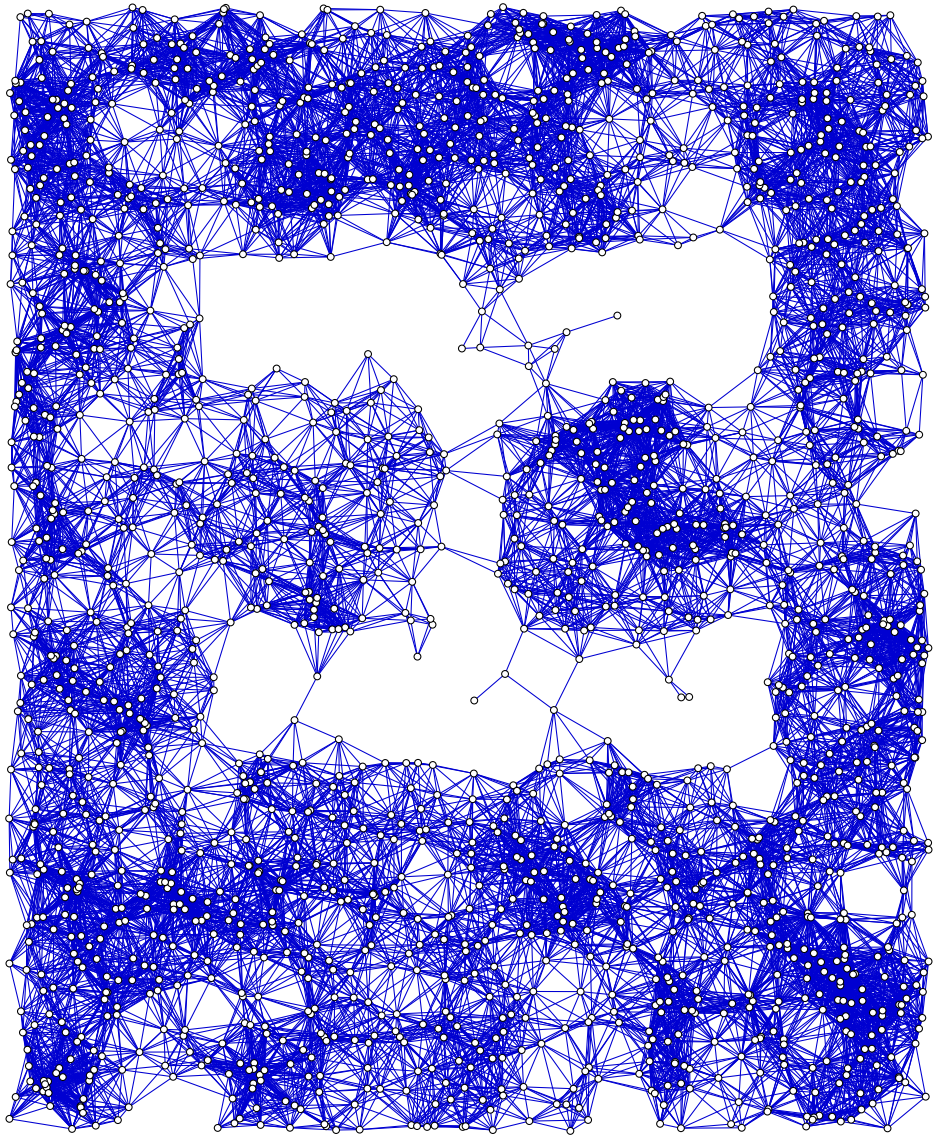
P2P Network (Gnutella)





Ad hoc network, Large Scale Internet ?





More networks

- Social networks
- Biological networks
- Dependencies in Linux packages
- Road networks

Scientific Foundations

- Graph theory
- Theory of distributed computing

Graph Theory

- Extremal graph theory
- Random graphs
- Structured graphs (de Bruijn, hypercube, ...)
- Graph decomposition (treewidth, treelength, ...)
- Metric properties (doubling, hyperbolic, ...)

Theory of Distributed Computing

- Locality issues (synchronous)
  - LOCAL and CONGEST models
  - Spanners
  - Informative labeling schemes
- Fault tolerance issues (asynchronous)
  - Shared memory and message passing models
  - Impossibility of consensus
  - Herlihy like hierarchies

Fundamental example

Question

- How much information is required to estimate $d(u, v)$?

Distance labeling

- Give labels $L(u), L(v)$ such that $d(u, v) \approx f(L(u), L(v))$.
- Size of labels vs accuracy of distance estimations?

Theorem [Slivkins'05]

- If d is a doubling metric then polylogarithmic labels allow to approximate distances within a factor $1 + \varepsilon$.

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GANG Achievements

For short

- Papers : PODC, STOC, FOCS, SODA, INFOCOM, ESA, ICALP, DISC,...
- A startup (OSEO Grant 2008)

Some facts

- Internet has doubling dimension ≈ 1.2.
- Internet has treewidth ≥ 80.
- Facebook graph has diameter 41.

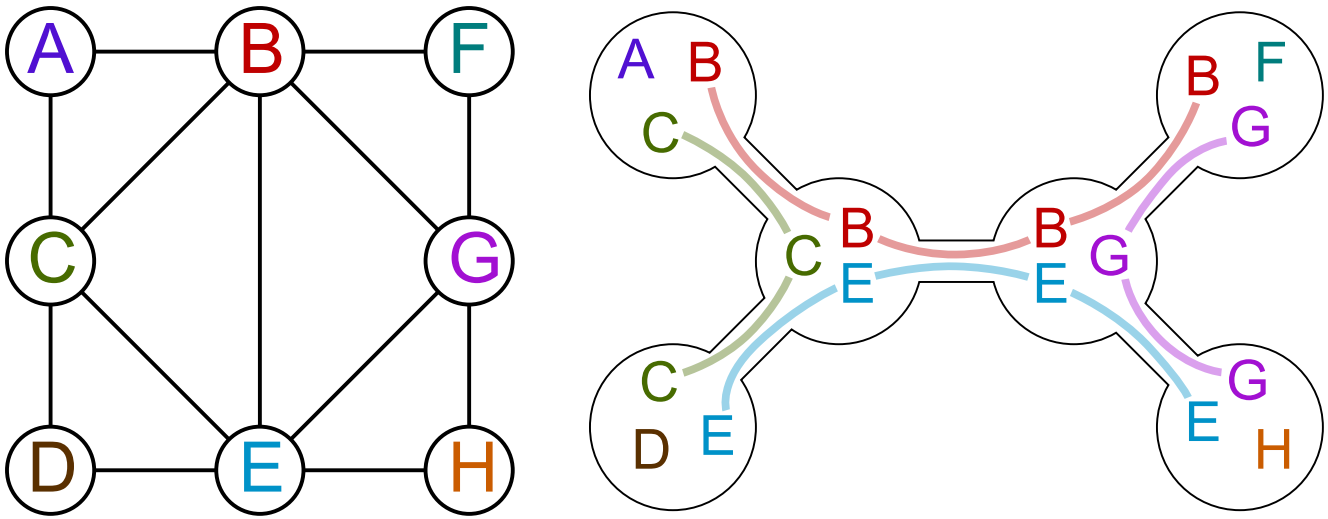
Objectives

- Large networks
- Decentralized content distribution
- Graph algorithms
- Distributed computing

Large Networks : Structure of Internet

- Internet delays as an Inframetric with low doubling dimension.
  [Fraigniaud, Lebhar, Viennot, INFOCOM'08]

- Treewidth and hyperbolicity of Internet.
  [de Montgolfier, Soto, Viennot, NCA'11]



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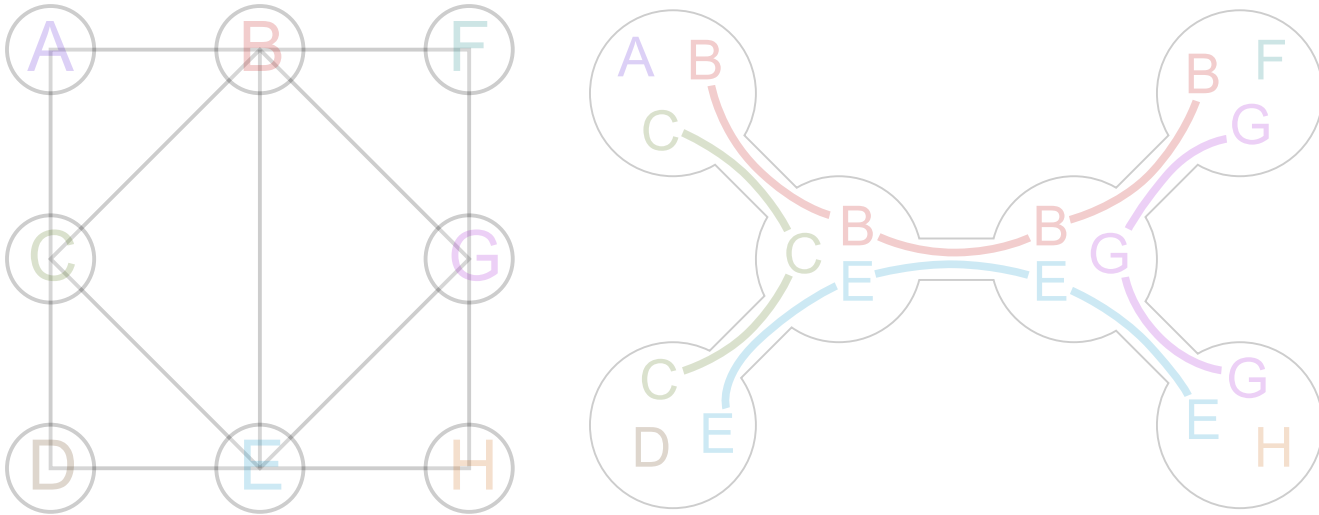

Estimating distances in Internet

Distance measure

- $d(u, v) = \mathrm{RTT}(u, v)$ (ping time from u to v)

Question

- Is this a doubling metric ?

Polynomial balls

- $B(u, r) = \{v : d(u, v) \leq r\}$
- Doubling dimension a :
  $B(u, 2r)$ can be covered by 2^a balls of radius r.
- Particular case : $|B(u, 2r)| \leq 2^a |B(u, r)|$.

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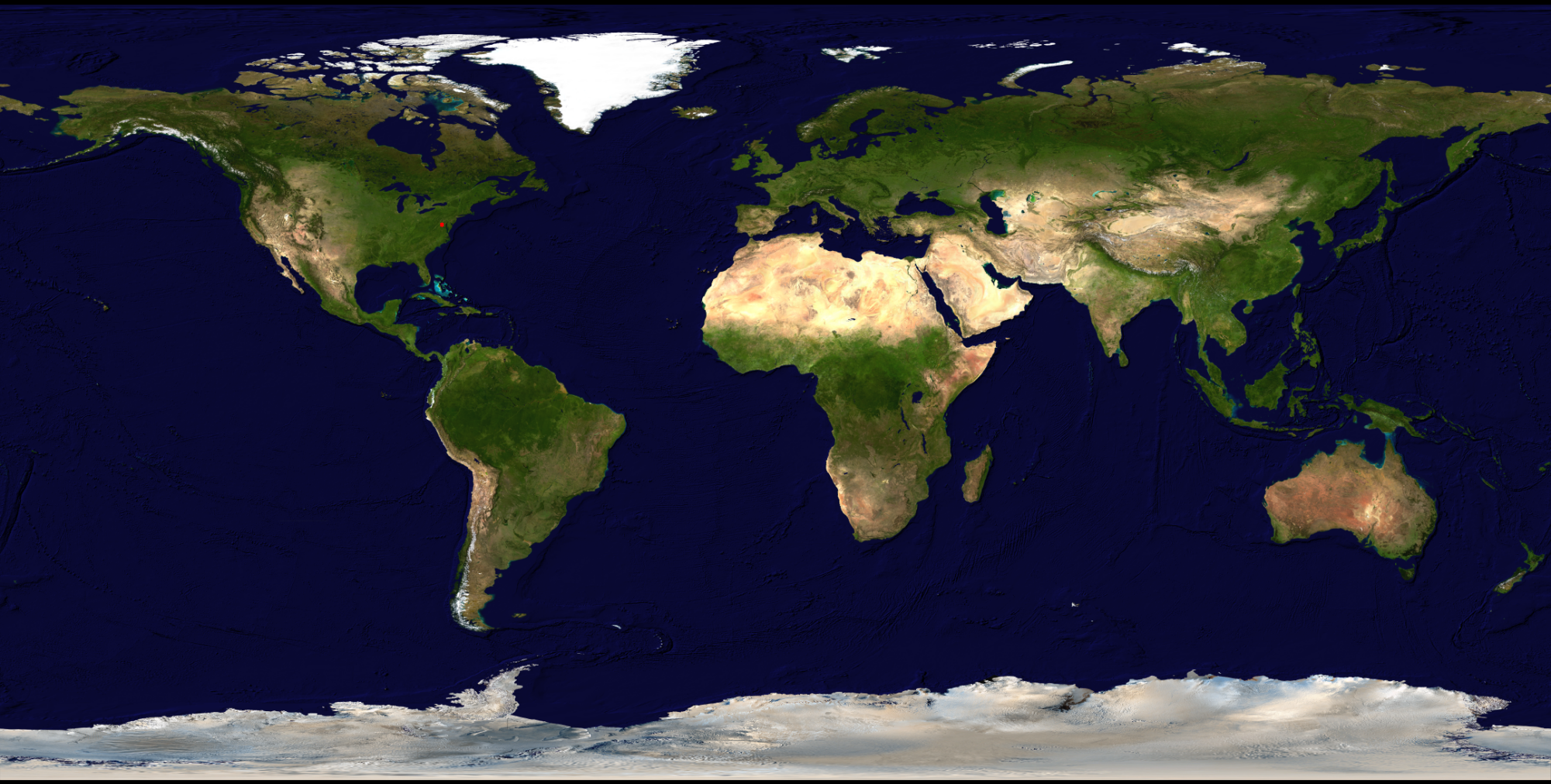
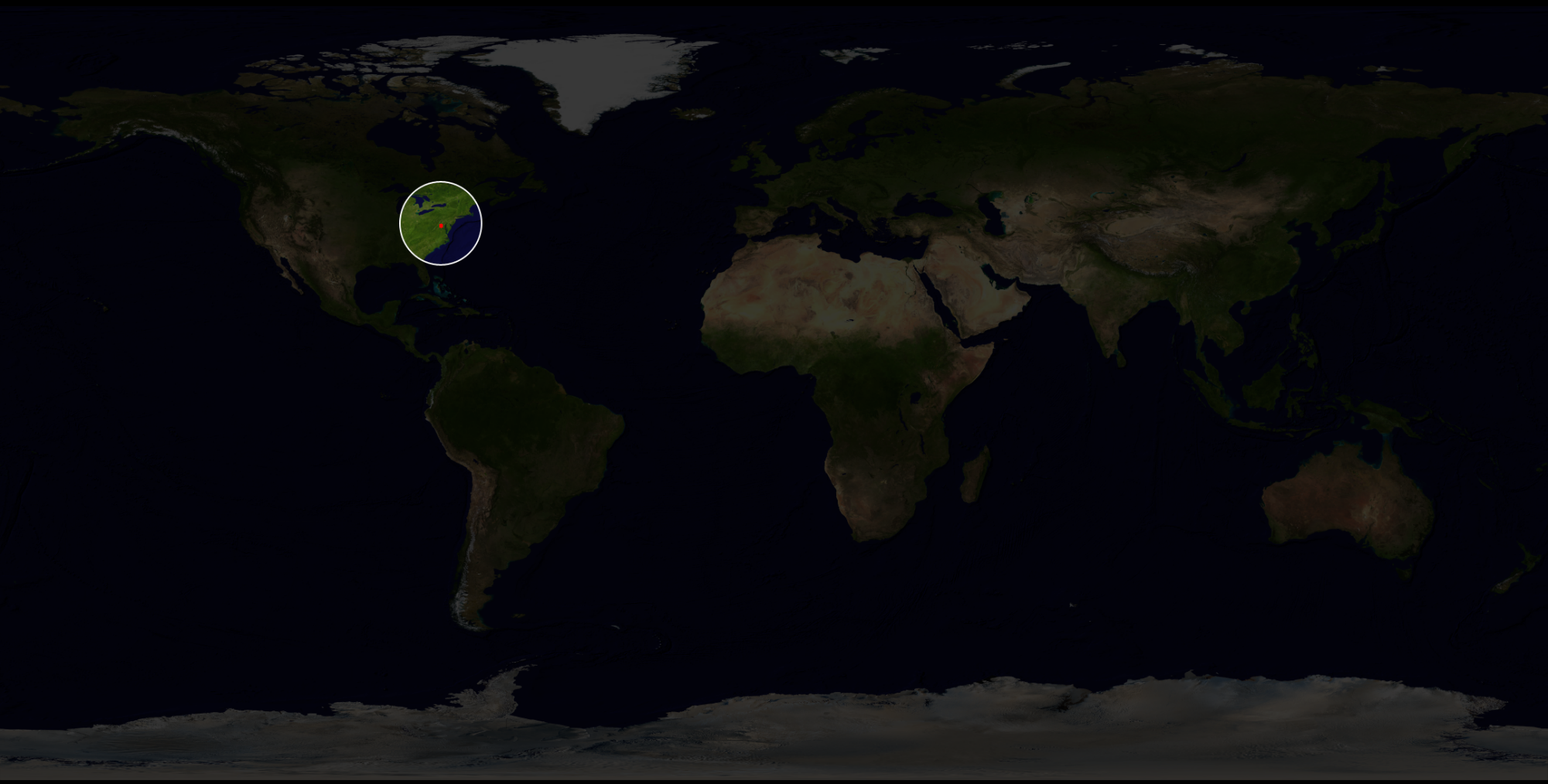
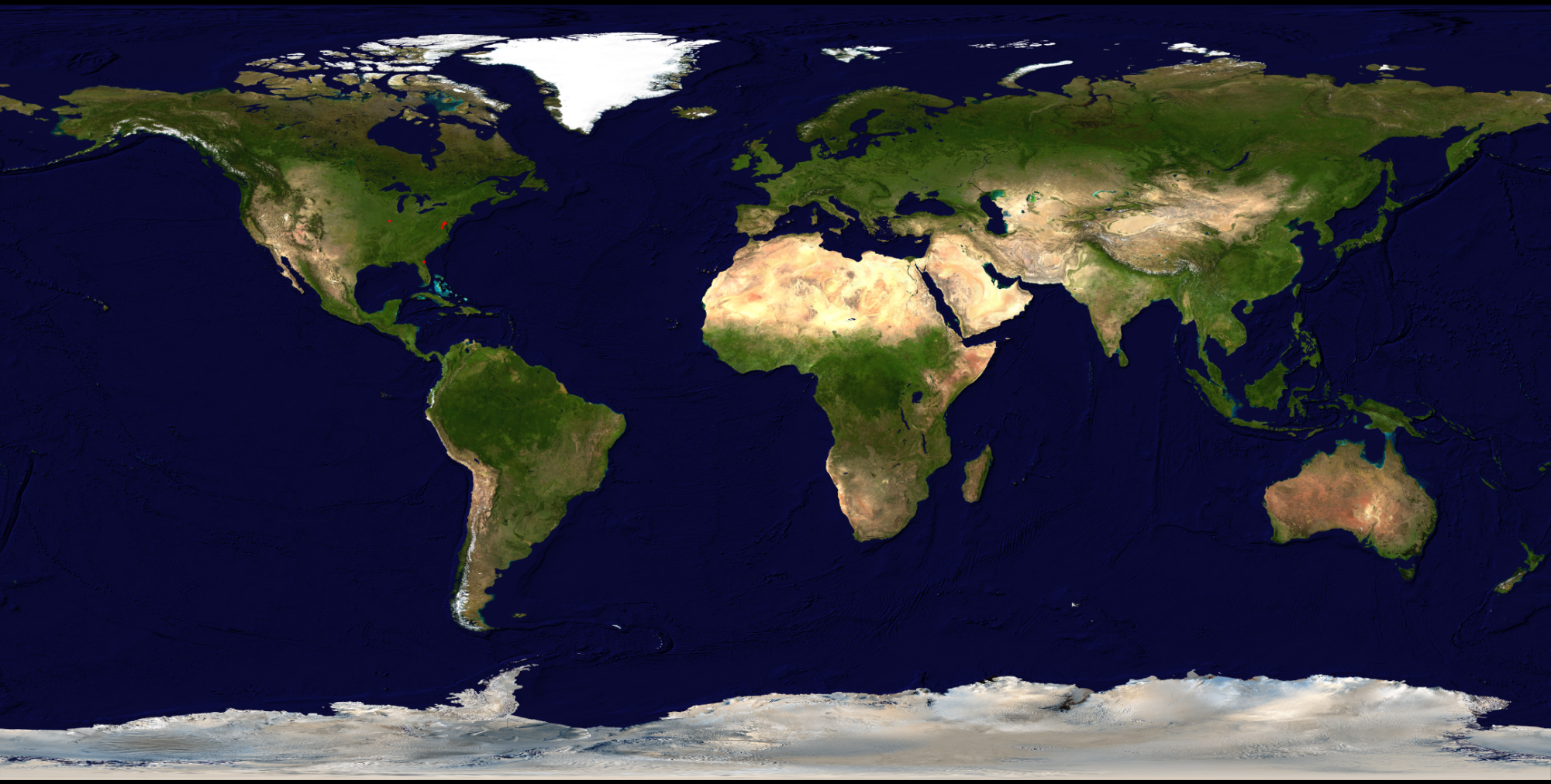
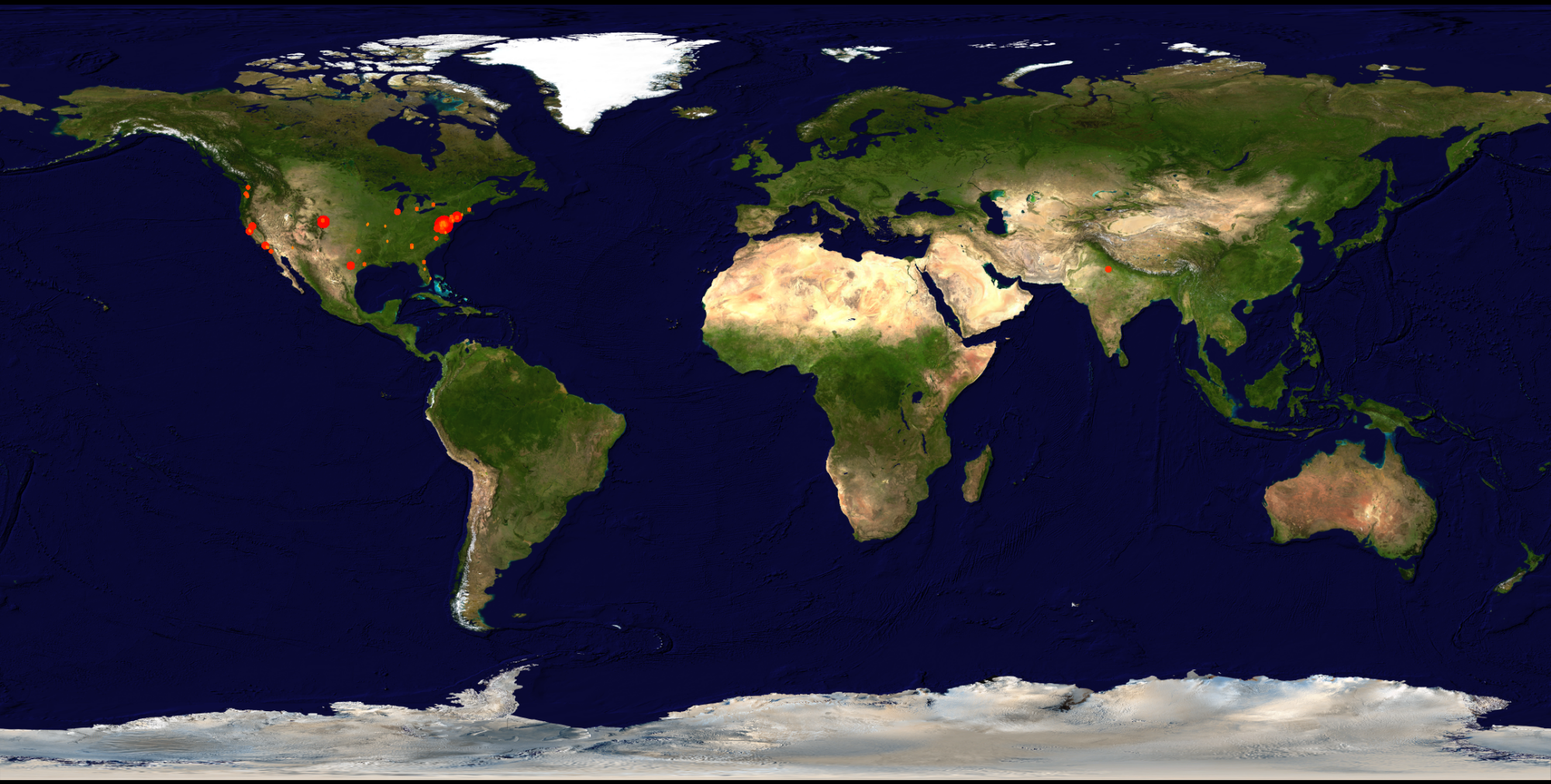




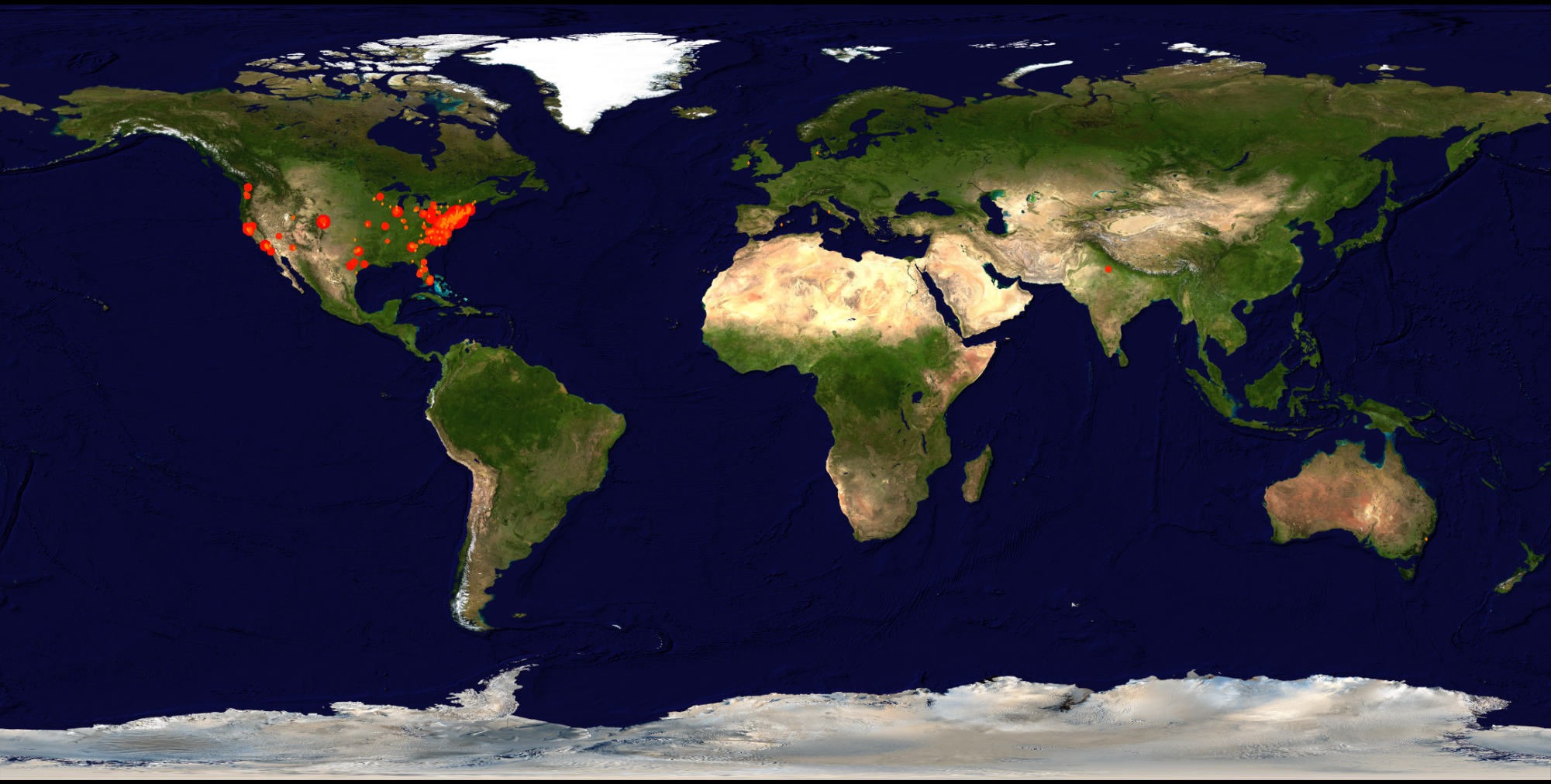
arin (2007/12)



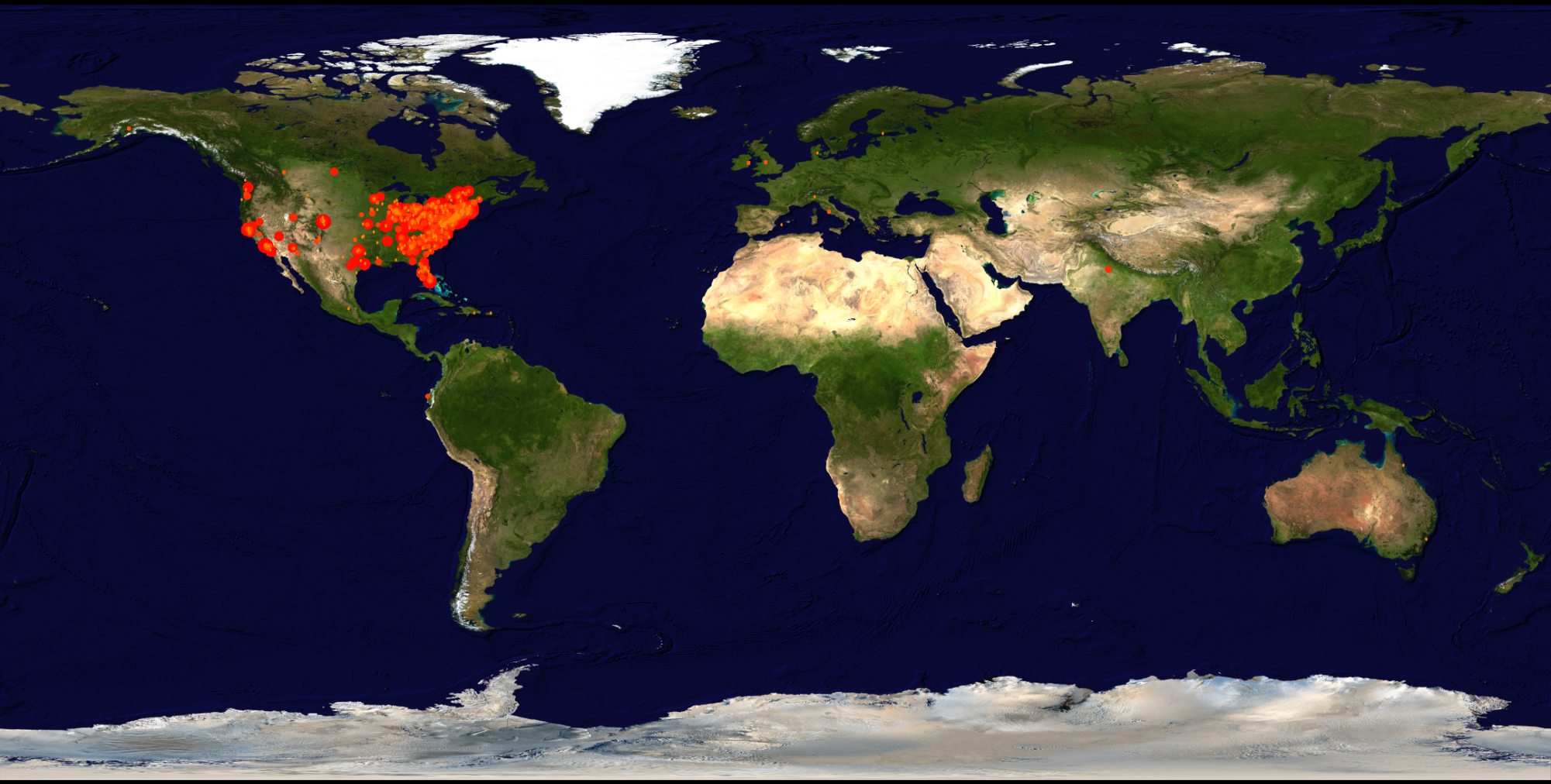
arin 4ms : 12 nodes



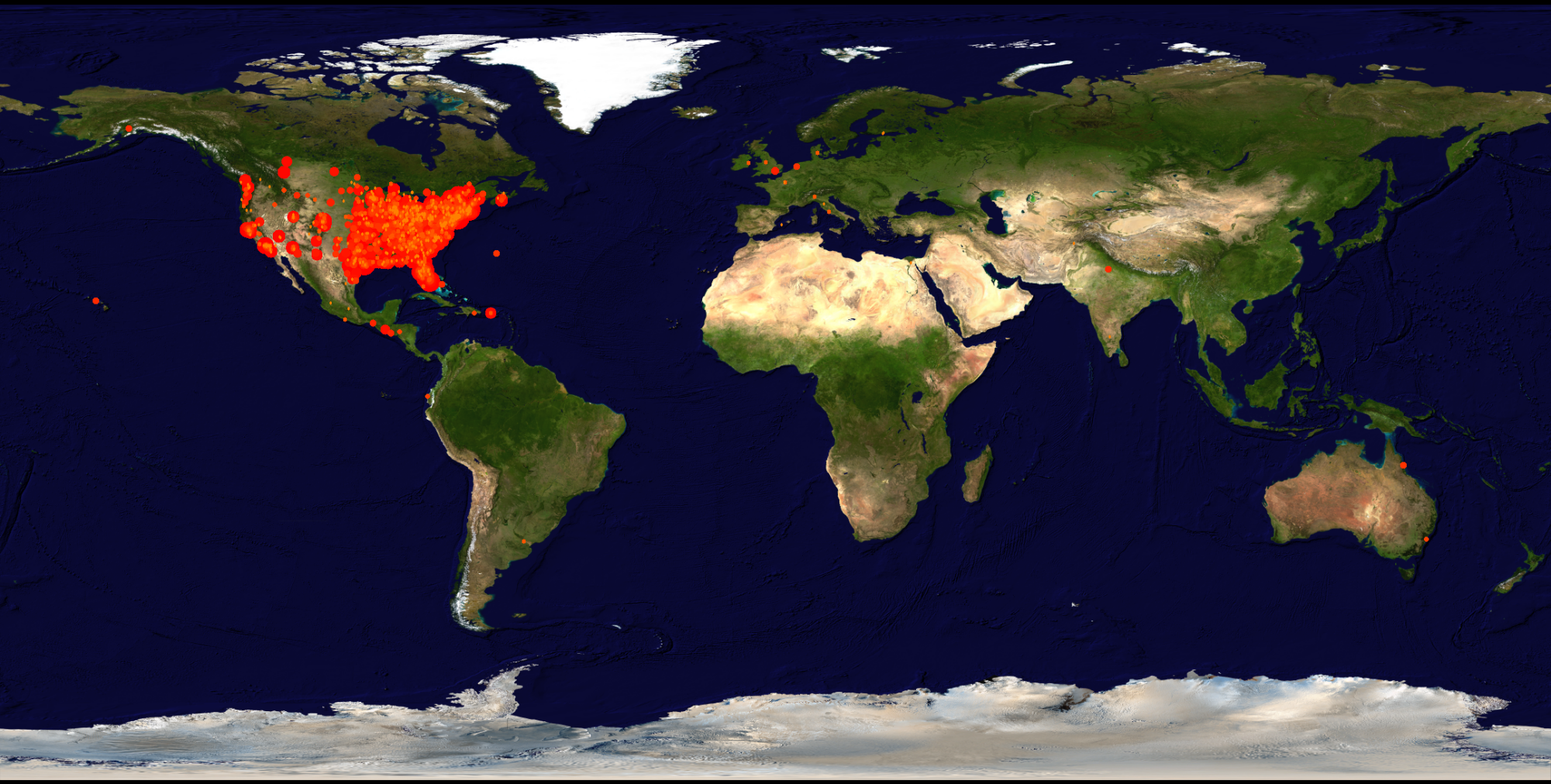
arin 8ms : 1706 nodes



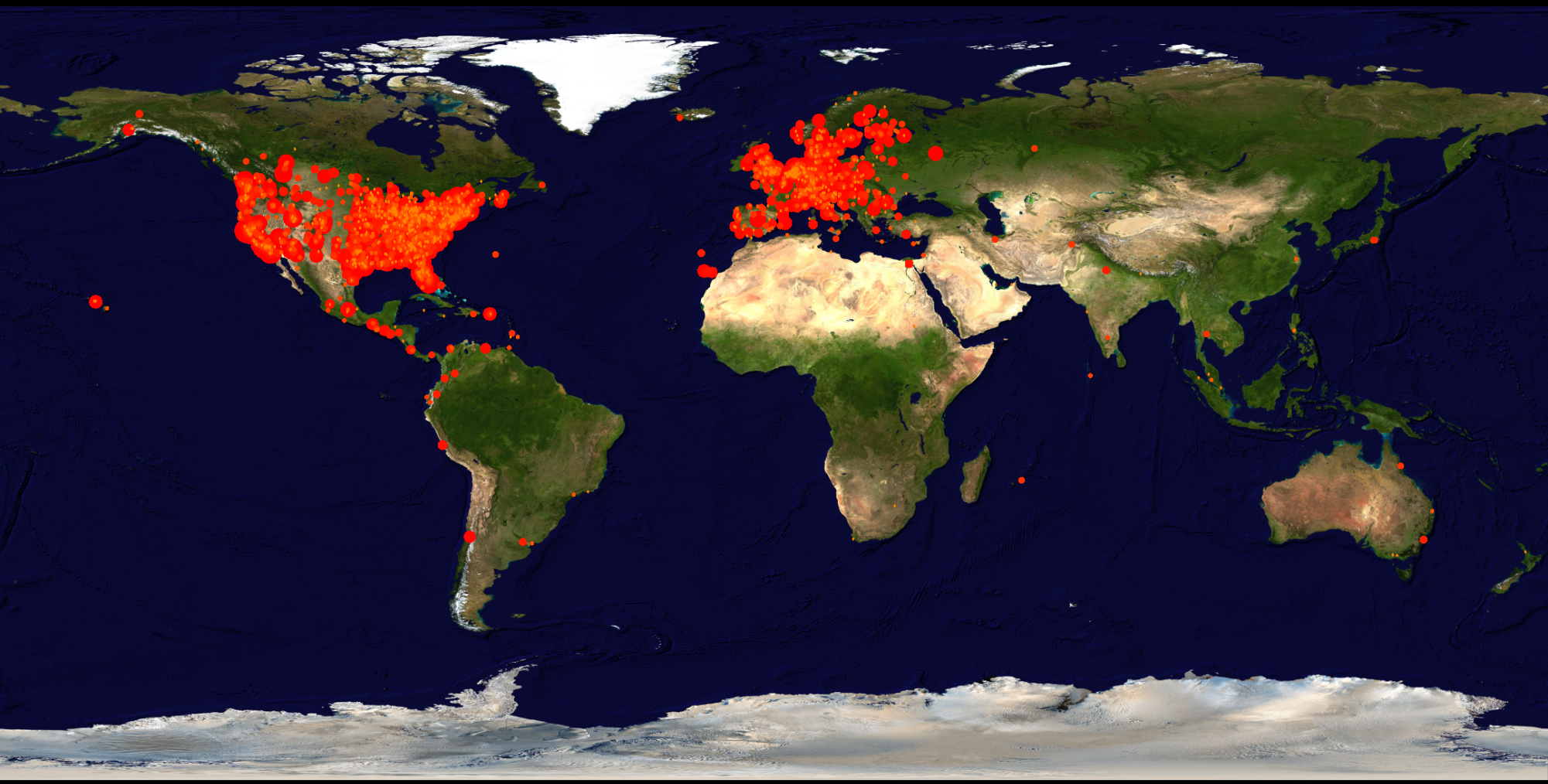
arin 16ms : 9046 nodes



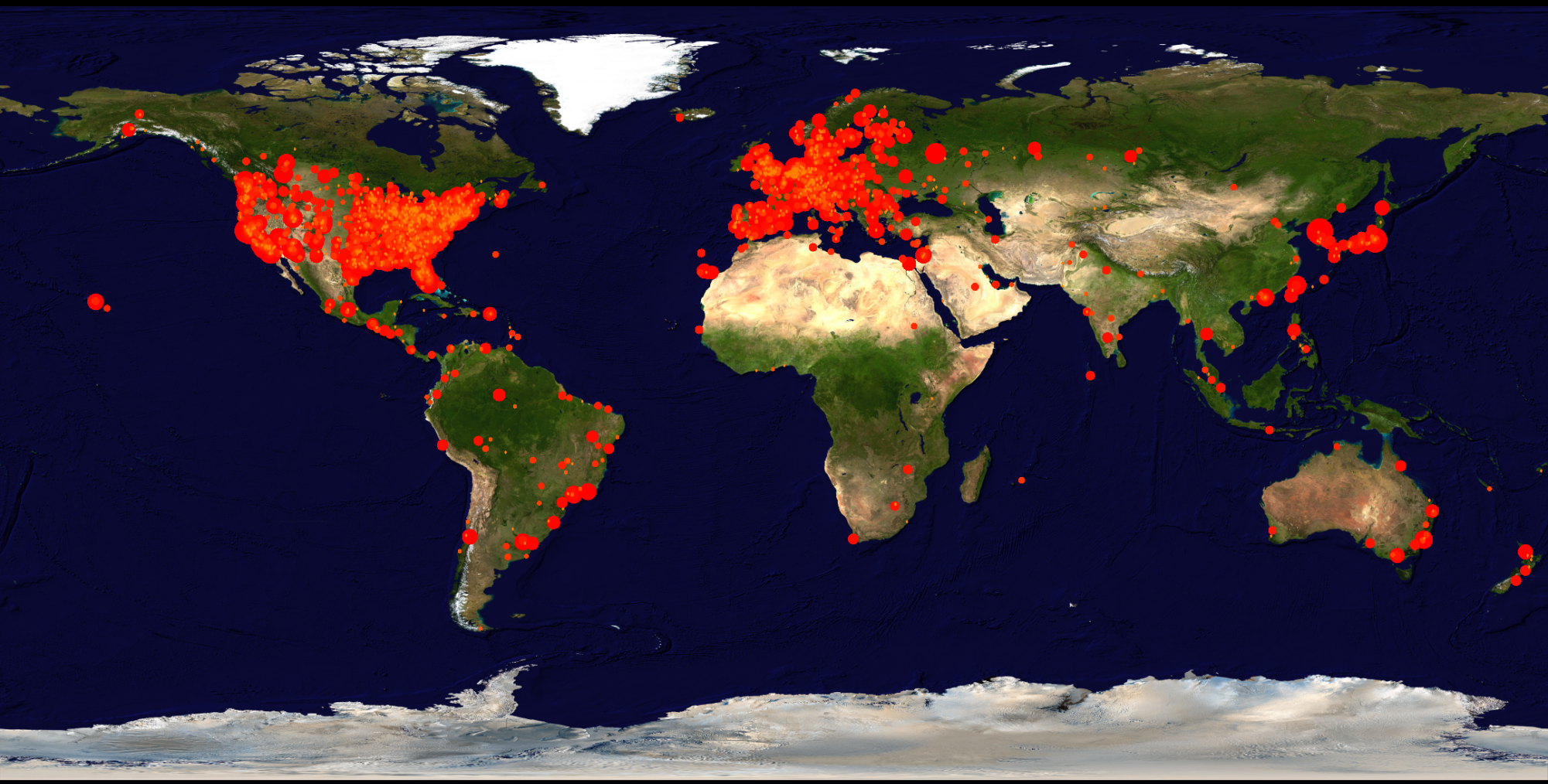
arin 32ms : 24861 nodes



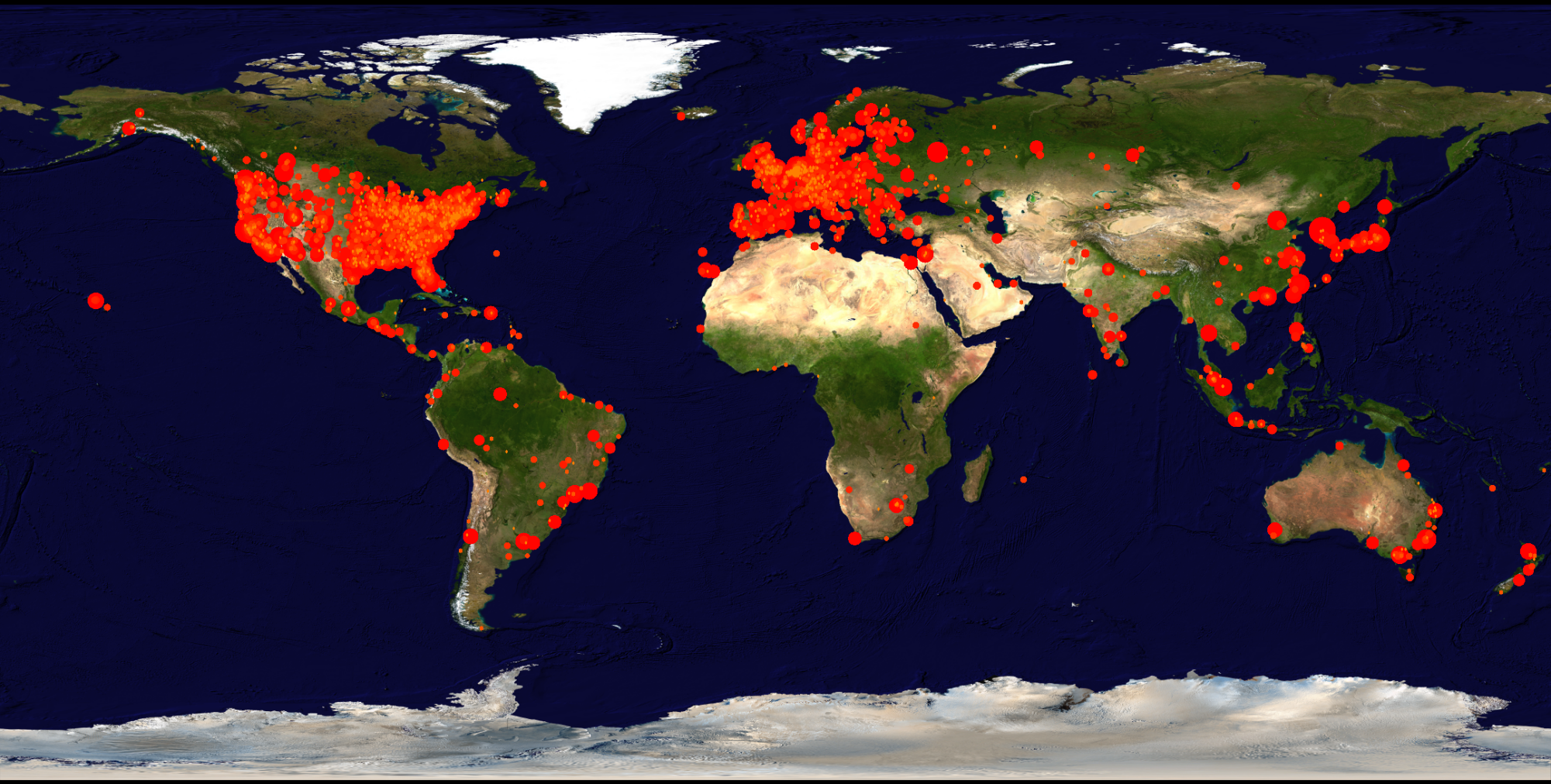
arin 65ms : 58180 nodes



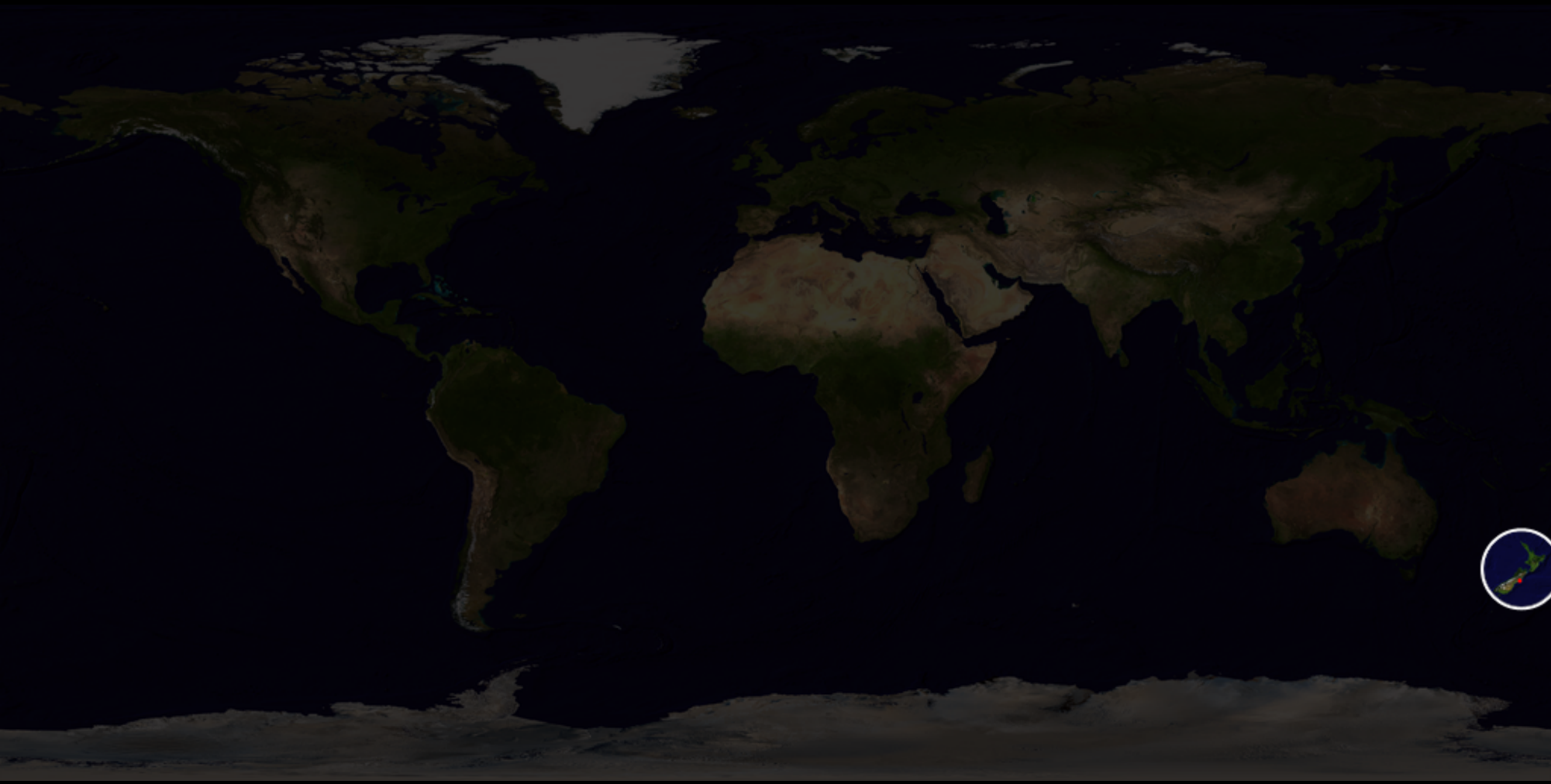
arin 130ms : 132309 nodes



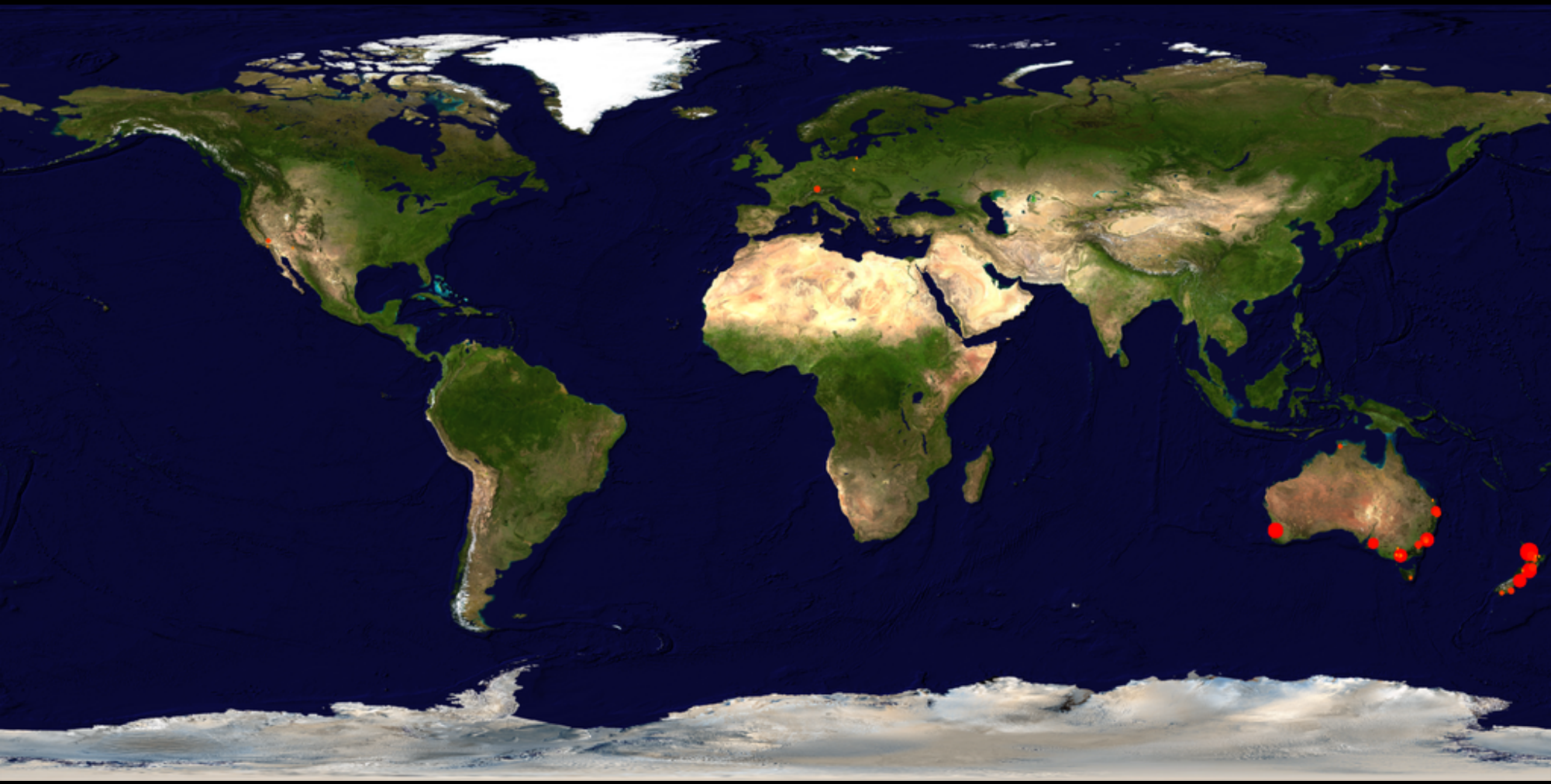
arin 260ms : 185487 nodes



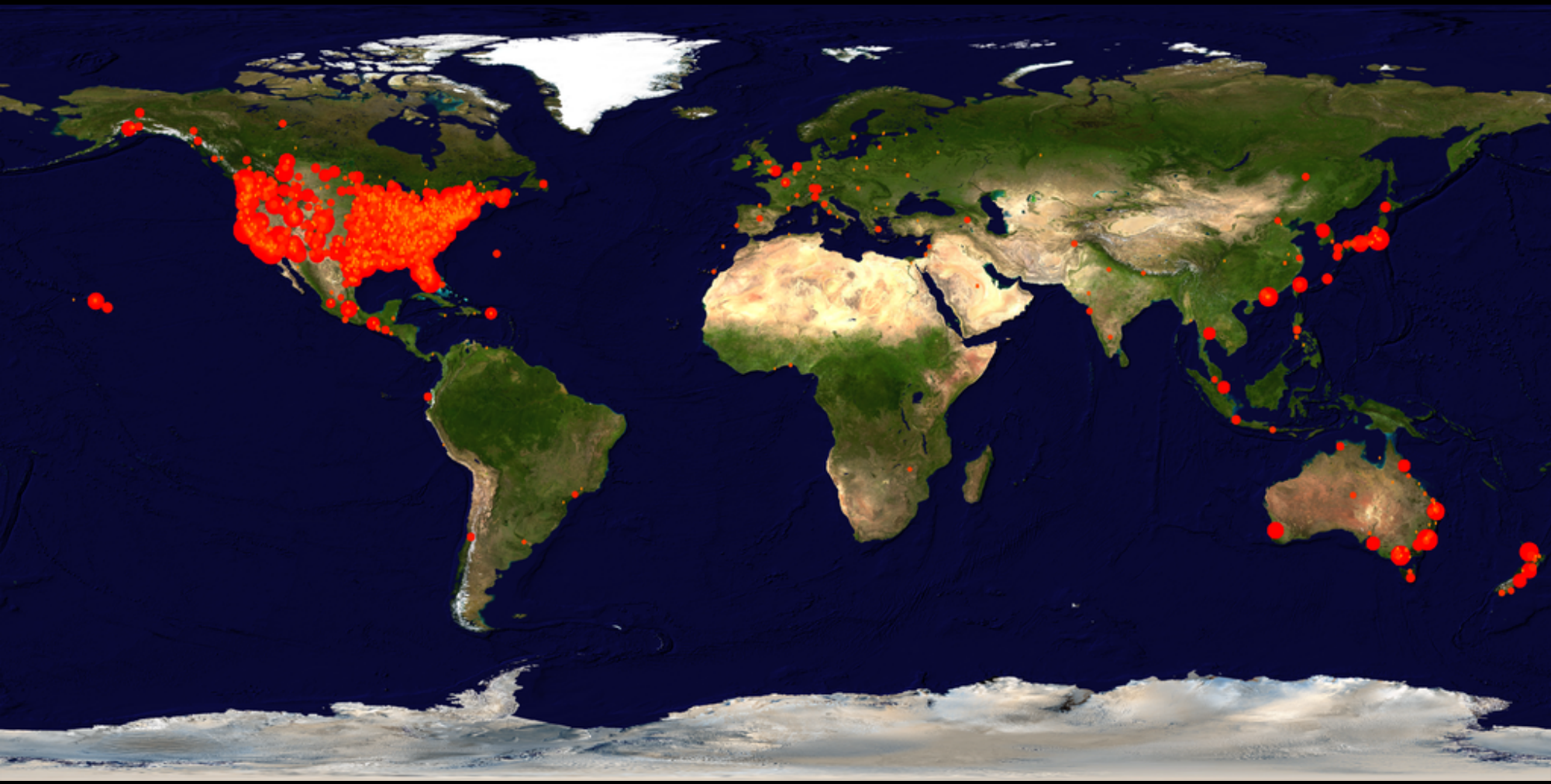
arin 520ms : 201891 nodes



ihug (2004/05)



ihug 130ms : 1921 nodes



ihug 260ms : 190161 nodes

Answer

No

- Not a metric : triangular inequality is often violated.
- No polynomial *growth* of balls.
- Doubling dimension appears to be high.

But

- ρ-inframetric : $d(u,v) \leq \rho \max\{d(u,w), d(w,v)\}$ ($\rho \approx 4$).
- Doubling' dimension $a \approx 1.2$:
  $B(u,r)$ can be covered by $O(r^{1.2})$ unit balls.

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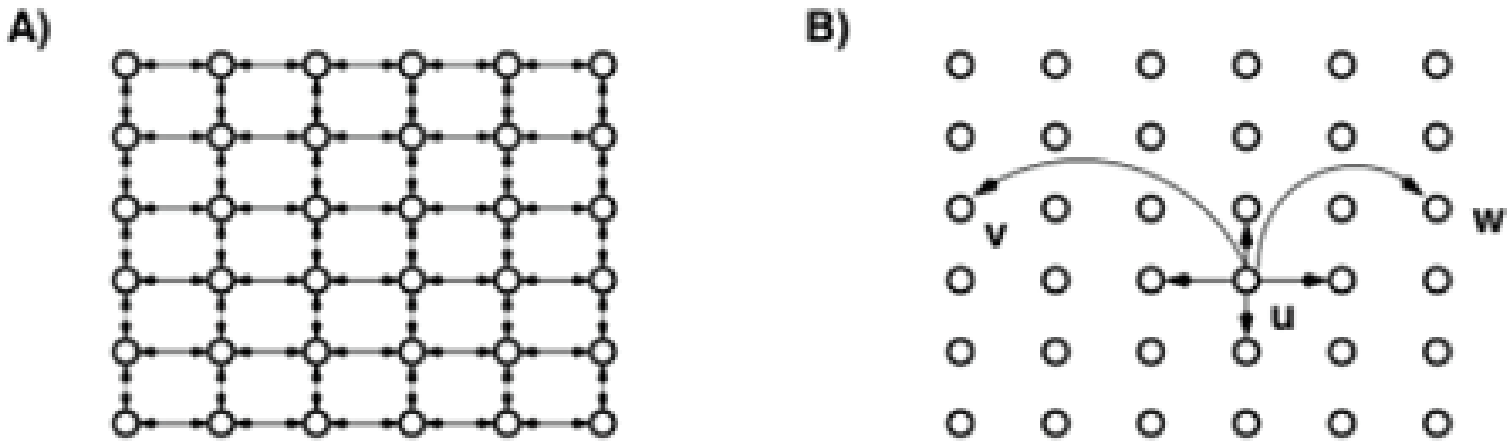
Large Networks : Social Networks

- Navigability of any graph : focus routing.
  [Fraigniaud, Giakkoupis, STOC'10]

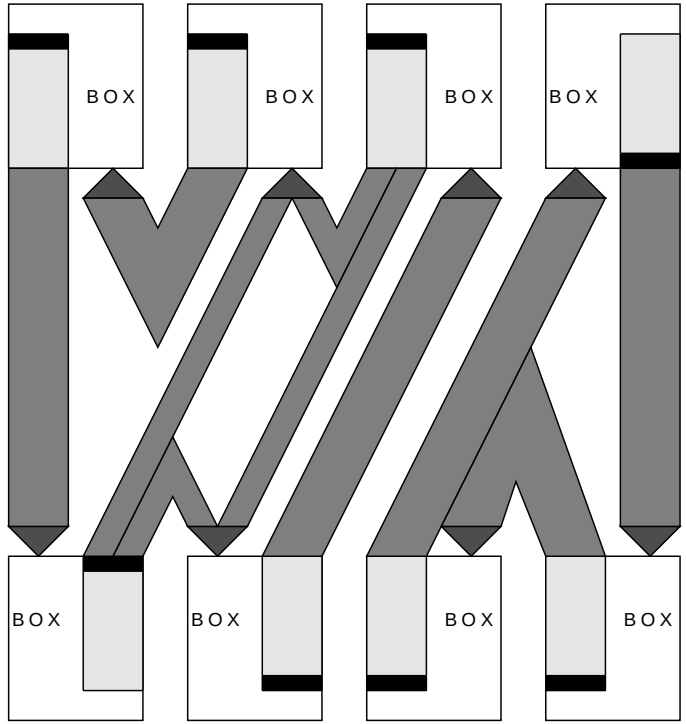
- Model for long range contacts : move and forget.
  [Chaintreau, Fraigniaud, Lebhar, ICALP'08]



Decentralized Content Distribution

- Epidemic live streaming.
  [Bonald, Massoulié, Mathieu, Perino, Twigg, SIGMETRICS'08]

- Distributed VoD with distributed catalog.
  [Boufkhad, Mathieu, de Montgolfier, Perino, Viennot, IPDPS'09]



Decentralized Content Distribution : Start-up

- Move&Play company : personal data sharing (OSEO creation-development grant 2008)
- Now CleverScale : CDN broker (7 persons)

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Graph Algorithms

- Diameter estimation via 4-sweep-BFS method.
  [Crescenzi, Grossi, Habib, Lanzi, Marino, ESA,TCS'11]

- Some graph problems in phylogenetics.
  [Thu-Hien To PhD thesis'11]

- Piecewise convex maximization problems
  [Fortin, Tseveendorj, J. Optim. Theory Appl.'11]

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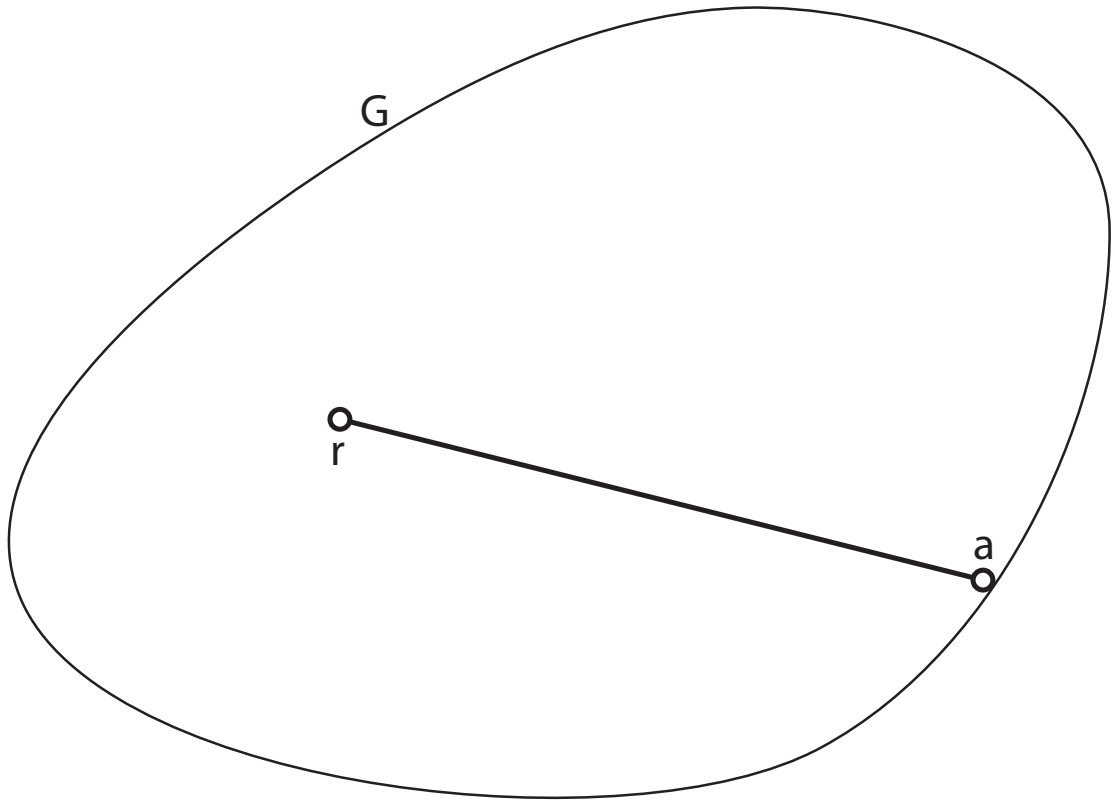

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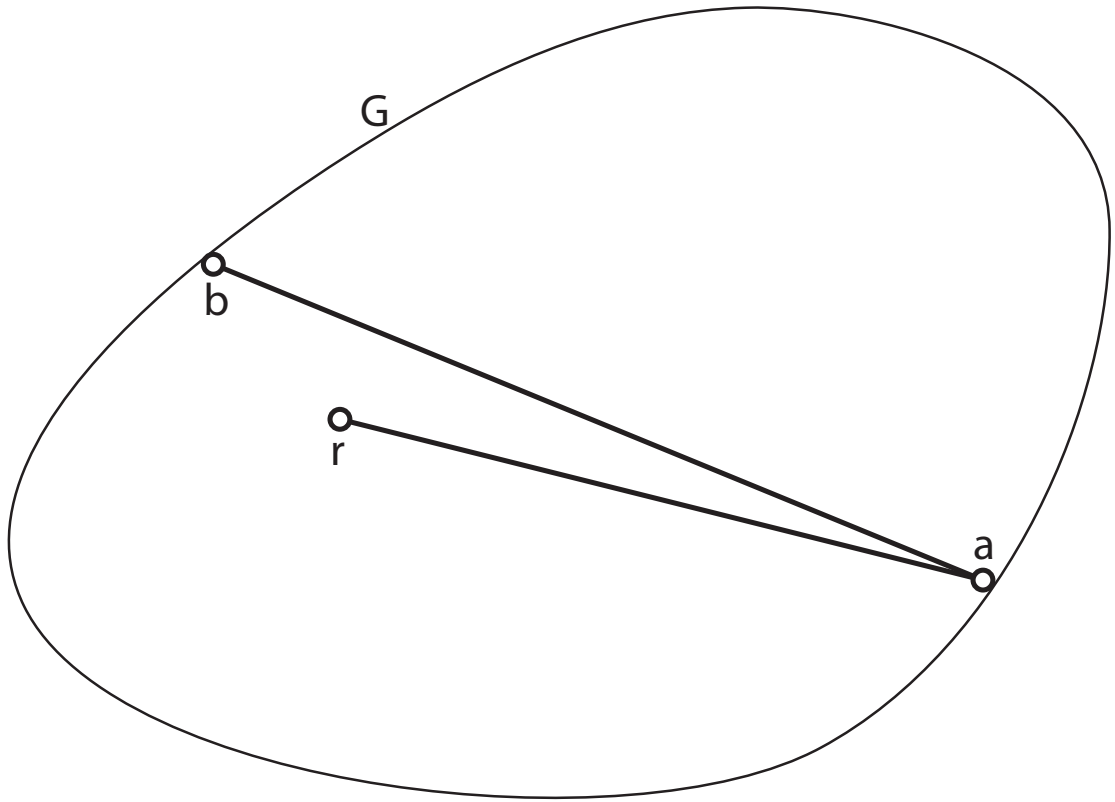

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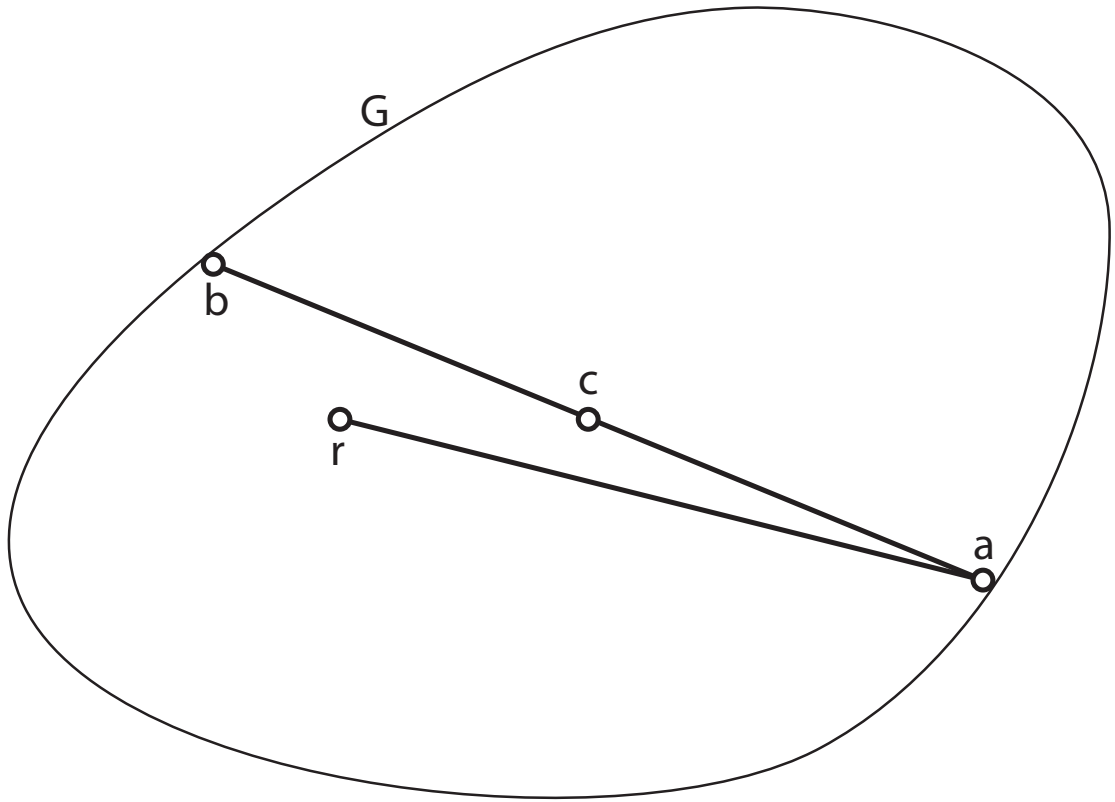

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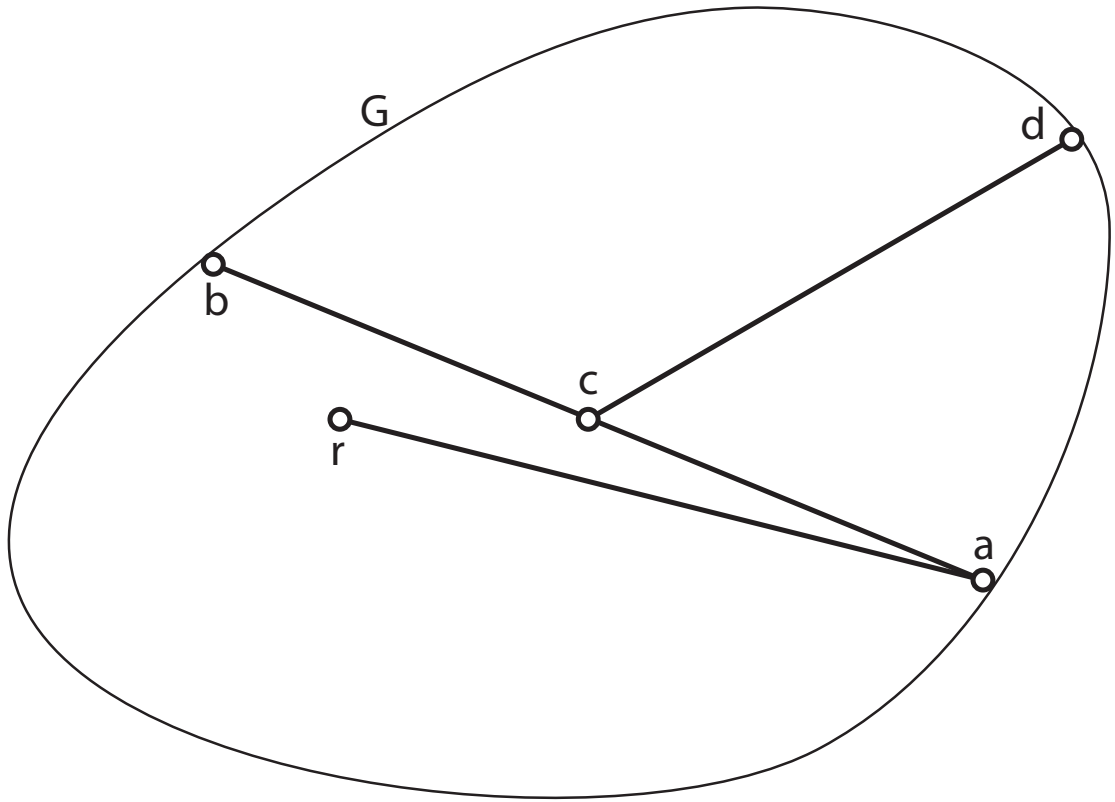

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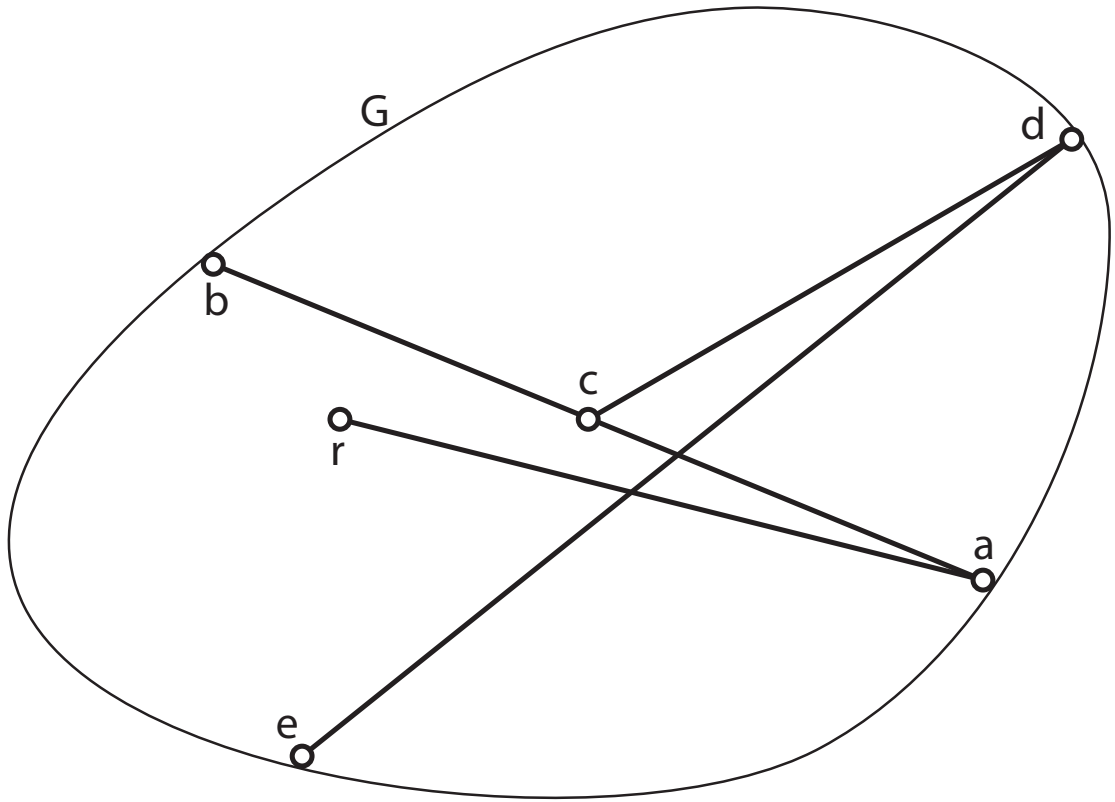

Distributed Computing

- Weakest failure detector.
  [Delporte, Fauconnier, Guerraoui, J. ACM'10]

- Class of problems that can be solved with combination of crashes, equivalence to k-set agreement.
  [Delporte, Fauconnier, Guerraoui, Tielmann, best paper DISC'09]

- Byzantine agreement with homonyms.
  [Delporte, Fauconnier, Guerraoui, Kermarrec, Ruppert, Tran-The, PODC'11]

Bringing Theory towards Practice

- Remote spanners (spanners for ad hoc networks).
  [Jacquet, Viennot, IPDPS'09]
- Multipath spanners (spanners for multipath routing).
  [Gavoille, Godfroy, Viennot, OPODIS'11]
- Distributed spanner computation.
  [Derbel, Gavoille, Peleg, Viennot, PODC'08]
- Compact routing and distance labeling for low hyperbolicity graphs.
  [Chepoi, Dragan, Estrellon, Habib, Vaxes, Xiang, Algorithmica'11]
- Getting rid of knowledge of n or other global parameters.
  [Korman, Sereni, Viennot, PODC'11]
- Using proof labeling schemes for autostabilizing MST.
  [Korman, Kutten, Masuzawa, PODC'11]

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Perspectives

Objectives

- Graph algorithms
- P2P approach to future content distribution
- Distributed computing

Graph Algorithms

- Study the power of graph searching on networks and graphs.
- Analyze some biological networks (clustering, compression, ...)
- Study approximations for consecutive ones property of matrices.

Graph Algorithms : Multi-sweep Methods

- Graph searching as a generic building block.
- Combine searches : the ordering produced by a search can be used to break ties in the next search.
- Explore searching procedures based on ordering properties : BFS, DFS, LexBFS, LexDFS, LexUp, LexDown,...

P2P Approach to Future Content Distribution

- P2P distribution vs content centric networking.
- Models where the bandwidth bottleneck is at the core of the network.
- P2P storage and caching allocation : interplay between CDNs, clouds and P2P.

Distributed Computing

- Explore distributed decision problems.
- Algorithms with oracles (failure detectors, a priori information).
- Non determinism in distributed computing.
- New computational paradigms such as distributed quantum computing.

Distributed Computing : Fault Tolerant Data Structures

- Estimate $d(u, v, F) = d_{G-F}(u, v)$.
- Fault tolerant spanners.
- Fault tolerant routing.
- Forbidden set labeling :
  $d(u, v, F) \approx f(L(u), L(v), \{L(x) : x \in F\})$.
- A new approach to amortized dynamic algorithms : add changes to F, recompute the data structure when F is too big.

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Thank You