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Optimal monotone forwarding policies in delay tolerant mobile ad-hoc networks

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ABSTRACT

We study fluid approximations for a class of monotone relay policies in delay tolerant ad-hoc networks. This class includes the epidemic routing and the two-hops routing protocols. We enhance relay policies with probabilistic forwarding, i.e., a message is forwarded to a relay with some probability p. We formulate an optimal control problem where a tradeoff between delay and energy consumption is captured and optimized. We compute both the optimal static value of p as well as the optimal time dependent value of p. We show that the time-dependent problem is optimized by threshold type policies, and we compute explicitly the value of the optimal threshold for some special classes of relay policies.

1. Introduction

In delay tolerant mobile ad-hoc networks, instantaneous connectivity is not needed any more and messages can arrive at their destination thanks to the mobility of some subset of nodes that carry copies of the message. A naive approach toward forwarding a message to the destination is to use epidemic routing in which any mobile that has the message keeps on relaying it to other mobiles that arrive within its transmission range and which do not yet have the message. This would minimize the delivery delay, albeit at the cost of inefficient use of network resources (in terms of memory used in the relaying mobiles and in terms of the energy used for transmitting multiple copies of the message).

The need for efficient use of network resources motivated the use of more economic forwarding schemes such as the two-hop routing protocols: the source transmits copies of its message to all mobiles it encounters, but the latter relay the message only if they meet the destination. Furthermore, timers have been proposed to be associated with messages stored at relay mobiles, so that after some threshold (possibly random) the message is discarded. The performance of the two-hop forwarding protocol along with the effect of the timers has been evaluated in [1]; the framework proposed there allows performance optimization by choosing the average timer duration.

In this paper, we introduce two alternative approaches for optimizing forwarding protocols. The first approach consists of forwarding a message to another relay with some probability p, where p can be optimized to meet some tradeoff between delay and resource utilization. The second optimization approach introduced is based on further allowing the forwarding probability p to vary in time. These two approaches are studied in this paper in conjunction with a wide class of monotone relaying schemes of which the epidemic routing and the two-hops routing are special cases (a precise definition of monotone relaying policies is postponed until Section 3).

In order to optimize p in the context of a general monotone relaying policy, we first introduce fluid approximations of the system dynamics. We then use tools from optimization and optimal control theory to come up with optimal static and

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dynamic choices for the parameter p. In the dynamic case, we establish the optimality of threshold type policies that use $p = 1$ up to some time t and then switch to the smallest possible value of p. We validate the fluid approximation through simulations, and compare it with the original discrete model. We illustrate through extensive numerical experimentation the benefits of our optimization approaches. For the special cases of epidemic routing and two-hop routing, we obtain explicit expressions for the performance measures corresponding to the optimal static and the optimal dynamic optimization problems. We compute in particular the optimal probability of successful delivery of the message by some time t for these forwarding schemes under the constraint that the energy consumption until time t is bounded by some constant $\mathcal{E}$.

The structure of the paper is as follows. Section 2 presents related work, and the one that follows introduces the model. We solve in Section 4 the static control problems and establish the structure of optimal dynamic control policies. We then consider in Section 5 the special cases of two-hop routing and epidemic routing, and compute the optimal policies in these two cases. In Section 6 we describe two interesting variants of the original problem: the first one applies to the case when the destination identity is not disclosed, the second one provides some results for the case when messages have associated a timer. Section 7 provides a validation of the fluid model through simulations. It further provides an extensive numerical investigation that illustrates the usefulness and power of our optimal control approach. Section 8 concludes the paper.

2. Related works

Delay Tolerant Networks (DTNs) have recently attracted increasing attention from the research community. The literature reports several results of real experiments on DTNs [2–4]. In [2], the DieselNet network was deployed over a wide urban area, using buses as mobiles. Also, the authors of [3] describe the use of human mobility to diffuse information through portable devices. In all these works, authors identify several technical problems; in particular, due to lack of persistent connectivity, one central issue of DTNs is routing, and applicable techniques depend on the knowledge on input variables such as contact times, traffic demands or memory occupation [5].

In the case of routing with zero knowledge [6,7], the problem is to deliver messages to destinations with high probability despite nonexistence of any *a priori* information on the encounter pattern of mobile devices. In this respect, due to its simple and robust implementation, one of the first proposed forwarding algorithms has been epidemic routing [8,9], also called controlled flooding [10]. Another proposed routing mechanism is the two-hop algorithm. Grossglauser and Tse proposed the two-hop routing algorithm in [11], with the main goal being to obtain a characterization of the capacity of mobile ad-hoc networks. The aim was to overcome the severe capacity limitations of static wireless networks [12]. The standard reference on the analysis of the two-hop relaying protocol is [13]. Fluid approximations and infection spreading models are used in [14].

One *leitmotiv* of message diffusion algorithms is how to trade off message delay for energy consumption, i.e., number of copies per delivered message. In this respect epidemic routing and two-hop routing stand at opposite ends. Epidemic routing has high delivery probability, but it floods the system with copies of the messages. Conversely, two-hop routing spreads messages at a much smaller pace, but the price paid is lower delivery probability. Other variants include the use of Time-To-Live (TTL) counters, probabilistic forwarding [10] and K-limited forwarding [1,15].

The control of forwarding has been addressed in the ad hoc networks' literature, e.g in [16] and [17]. In [16], the authors describe an epidemic forwarding protocol based on the *susceptible-infected-removed* (SIR) model [14]. The authors of [16] show that it is possible to increase the message delivery probability by tuning the parameters of the underlying SIR model. In [17] a detailed general framework is proposed in order to capture the relative performance of different self-limiting strategies.

Finally, the authors of [18] argue that even in sparse DTNs, finite bandwidth, scheduling and interference affect the performance of epidemic routing; in this regard, the performance figures of this work should be viewed as upper bounds.

Novel contributions

As indicated before, we consider in this paper sparse mobile ad-hoc networks. In these networks, mobility is the engine that permits network-wide information diffusion. As compared with the existing literature, this paper makes several original contributions. Typically, the objective is to maximize the fraction of delivered messages and to minimize the latency between source and destination. In the paper we provide a formulation rooted in optimization, which relates explicitly the energy expenditure, i.e. the number of copies, to the delivery probability within a given deadline.

From an algorithmic standpoint, the fundamental result derived in this paper, using tools of optimal control theory, is that *optimal forwarding policies are threshold-type policies* (see Definition 4.1). It then follows that static control policies, i.e., probabilistic forwarding with constant probability, are suboptimal.

We note that the above mentioned results apply to all monotone forwarding strategies, which include several interesting cases for sparse ad-hoc networks, such as epidemic routing and two-hop routing. Further, the practical implications of this result should be apparent, since the implementation of threshold policies is rather straightforward.

3. The model

Consider a network of N mobile nodes. The time between contacts of any two nodes is assumed to be exponentially distributed with parameter λ. The validity of this model has been discussed in [13], and its accuracy has been shown for a number of mobility models, such as random walk, random direction, and random waypoint.

We assume that the message that is transmitted is relevant for some time τ, and that there is no feedback that allows the source or other mobiles to know whether the message has made it successfully to the destination within the allotted time window τ or not.

A mobile terminal is assumed to have a message to send to a destination node. We focus in this paper on a class of so called *monotone relay strategies*. A relay strategy is said to belong to this class if the following holds:

- The number of nodes that contain the message does not decrease in time during the time τ,
- The number $\widetilde{X}(t)$ of nodes, not including the destination, that contain the message at time t constitutes a Markov chain.

Example 1: Epidemic routing

At each encounter between a mobile that has the message and another one that does not, the message is relayed to the one that does not have it. This is a monotone relay strategy.

Example 2: Two-hop routing

At each encounter between the source and a mobile that does not have the message, the message is relayed to that mobile. If a mobile that is not the source has the message and it is in contact with another mobile, then it transfers the message if and only if the other mobile is the destination node. This is a monotone relay strategy.

Example 3: Adding timers

Consider either the epidemic routing or the two-hop routing. In order to avoid saturation of the buffers of relay mobiles, a relay mobile that receives a message activates a TTL timer which is exponentially distributed with parameter μ. $\widetilde{X}(t)$ is indeed a Markov chain but it is not necessarily monotone non-decreasing. Thus strategies that integrate TTL timers may not be monotone relay strategies. We will discuss the implication of this result later in Remark 3.1.

As it is customary in the literature, we will often refer to nodes having a copy of the message as *infected* nodes.

3.1. The control

Let $\{T_n\}$ be the sequence of instants when an encounter takes place between two mobiles. Only at these time instants may the state $\widetilde{X}$ change.

The actual system dynamics is obtained by combining the relay strategy with the control. When two nodes meet at a given time T_n, one of them has a copy of the message and the other one does not, and under the relay strategy protocol, the message is transmitted to the other mobile. Such a time instant, T_n, is called a forwarding opportunity.

A natural way to optimize the system (with respect to some objectives that will be introduced shortly) is to control the forwarding probabilities of messages. We assume that each message contains a time stamp that shows when it was generated (so that it can be deleted at all nodes that have it when it becomes irrelevant, τ time units later). We shall consider two optimization approaches:

- *Static approach:* Each time there is a forwarding opportunity, forwarding of the message is done with a constant probability c.
- *Dynamic approach:* Each time a mobile has a forwarding opportunity, it checks the time t that has elapsed since the message generating time and it forwards the message with some probability $u(t)$.

In both cases we shall assume that the forwarding probabilities can take any value within an interval $[u_{\min}, 1]$, where $u_{\min} \geq 0$. Notice that some protocols may require $u_{\min} > 0$: this might be the case when distributed mechanisms are employed in order to track the cooperation level of users or to create reputation measures in a distributed way [19]. Further mechanisms which require a minimum amount of message forwarding are tit-for-tat mechanisms used to enforce cooperation in ad hoc networks [20].

3.2. Fluid approximations

We introduce here the fluid models used in the rest of the paper. The approximation of Markov chains through differential equations is a well known technique, see for example [21] for a survey. In the literature, the usage of fluid approximations is a standard tool to model epidemic forwarding [14,22,23]. The approximation holds tight for large populations of nodes; more precisely the sample paths of the Markov chain $\widetilde{X}$ are known to converge in probability to the solution of the limit differential equation, which represents the expectation of the number of copies of the message [21].

Uncontrolled dynamics. Let $\overline{X}(t)$ be the fraction of the mobile nodes that have at time t a copy of the message, given $\overline{X}(0) = z$. (It includes the source node and thus $\overline{X}(0) \geq 1/N$.) We assume that $\overline{X}(t)$ grows at a rate determined by the following differential equation:

$$\frac{\mathrm{d}\overline{X}(t)}{\mathrm{d}t} = f(\overline{X}(t)) \tag{1}$$

where f is assumed to be strictly positive.

Controlled dynamics. Let $X(t)$ be the fraction of the mobile nodes that have at time t a copy of the message. We assume that $X(t)$ grows at a rate determined by the following differential equation:

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = u(t)f(X(t)) \tag{2}$$

where $u(t) \in [u_{\min}, 1]$, $u_{\min} \geq 0$. u has the meaning of a control that is used to slow down the growth rate of the number of copies of the message in the network. We will occasionally denote the solution to (2) by $X(t; u)$ to emphasize its dependence on u.

$X(t)$ coincides with $\overline{X}(t)$ when $u(t) = 1$. In the special case where $u(t)$ is a constant, $u(t) = p$, the solution of (2) satisfies:

$$X(t) = \overline{X}(pt). \tag{3}$$

Hence, in this case, $X(t)$ has the same dynamics as $\overline{X}(t)$ but at a slowed time-scale.

Next, we write the fluid approximation for the probability distribution of the delay T_d, denoted by $D(t) := P(T_d < t)$. Based on [24, Appendix A], we have

$$D(t) = 1 - (1 - D(0)) \exp\Big(-N\lambda \int_{s=0}^{t} X(s)\mathrm{d}s\Big), \tag{4}$$

where $D(0) = z$ accounts for the probability that the destination is not among the nodes that possess the message at time 0. The expression in (4) derives from the differential equation in the form

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}D(t) &= -\lim_{h\to 0} \frac{\mathbb{P}\left[T_d > t+h\right] - \mathbb{P}\left[T_d > t\right]}{h} \\ &= \lim_{h\to 0} \frac{\mathbb{P}\left[T_d > t\right] - \mathbb{P}\left[T_d > t+h\right]}{h} \\ &= N\lambda\, X(t)\left[1 - D(t)\right], \end{aligned} \tag{5}$$

which is separable and integrates as

$$\int_{D(0)}^{D(t)} \frac{\mathrm{d}D}{1-D} = N\lambda \int_{0}^{t} X(s)\mathrm{d}s, \tag{6}$$

from which (4) follows directly.

Denote by $\mathcal{E}(t)$ the energy consumed by the whole network for transmitting and receiving during the time $[0, t]$. It is proportional to $X(t) - X(0)$ in view of our assumption that the message is transmitted only to mobiles that do not have it, and thus the number of transmissions of the message during $[0, t]$ plus the number of mobiles that had it at time zero equals the number of mobiles that have it. We thus have $\mathcal{E}(t) = \varepsilon(X(t) - X(0))$. Notice that ε represents the energy to transmit and receive a copy of the message.[1]

Remark 3.1. If TTL timers are added as discussed in Section 3, or more generally, when the nodes that have a copy of the message may loose it and may thus later receive it again, the energy spent for transmission until time t can be larger than $\varepsilon(X(t) - X(0))$. Thus a constrained optimization problem where we place a constraint on the transmission energy does not translate anymore into a constraint on the final state. Such cases will therefore be introduced later, in Section 6.2.

4. Optimal control for the fluid model

Our goal is to maximize $D(\tau)$. In view of (4), this is equivalent to maximizing

$$J(z, u) := \int_{0}^{\tau} X(r; u)\mathrm{d}r$$

for an initial state z, where $X(r; u)$ is the state trajectory under a control u (dependence on u will be suppressed in most of the development below).

On the other hand, we also would like to keep $\mathcal{E}(\tau)$ small.

Let $\sigma(x, z) := \overline{X}^{-1}(x + z)$ given $\overline{X}(0) = z$, which is the time elapsed until x extra nodes (in addition to the initial z ones, both viewed as a fraction of the total number of nodes) receive the message in the uncontrolled system ($p = 1$). Notice that $\sigma(x, z)$ is a function of both x and z; in the following, for the sake of notation, unless misleading, we will refer simply to $\sigma(z)$.

[1] It should also be noted that we neglected here the term related to the idle mode power consumption: in fact, it can be viewed as a linear term in τ, and as such it is not accounted for in our optimization framework.

4.1. Time change

In the following we formalize the fact that the controlled state evolves as a slower version of the uncontrolled one, and the control u can be interpreted as the slowing factor.

Let

$$ds = u(t)dt, \quad s(0) = 0. \tag{7}$$

Then

$$\frac{d\overline{X}(s(t))}{dt} = \frac{d\overline{X}(s)}{ds} \times \frac{ds(t)}{dt} = f(\overline{X}(s(t)))\, u(t) \tag{8}$$

with $\overline{X}(0) = X(0)$. $\overline{X}(s(t))$ thus satisfies (2), and we conclude that

$$X(t) = \overline{X}(s(t)). \tag{9}$$

Remark 4.1. (i) In the special case when a static policy $u(t) = p$ is used then we have $s(t) = pt$ so that $X(t) = \overline{X}(pt)$.
(ii) Since $u(t) \le 1$ it follows from (7) that for all $t \ge 0$, $s(t) \le t$ and hence $X(t) \le \overline{X}(t)$.
(iii) We can deduce from what we have so far that the trajectory $X(t)$ under any u can be expressed as one under $u_{\max}$ at an earlier point in time provided that the initial state is the same at time zero. It will also turn out to be useful to relate $X(t)$ to the trajectory under $u_{\min}$ keeping the terminal point at time τ the same. More precisely, we define $\underline{X}(t, \xi)$ to be the solution of the dynamics

$$\frac{dx(t)}{dt} = u_{\min} f(x(t))$$

with as terminal condition $x(\tau) = \xi$. Define now instead of (7) $d\hat{s} = u_{\min} dt$, which gives $\hat{s}(t) = \tau + (t - \tau)u_{\min}$. Then,

$$\frac{dX(\hat{s}(t))}{dt} = \frac{X(\hat{s})}{d\hat{s}} \times \frac{d\hat{s}(t)}{dt} = f(X(\hat{s}(t)))u_{\min}.$$

We conclude that

$$X(\hat{s}(t)) = \underline{X}(t, \xi)$$

where $\xi = X(\tau)$. This time, however, $\hat{s}(t) \ge t$ for all $t \le \tau$. We thus conclude that $X(t) \le X(\hat{s}(t)) = \underline{X}(t, \xi) \le \underline{X}(t, z + x)$.

4.2. Optimal static control

Theorem 4.1. *Consider the problem of maximizing $D(\tau)$ subject to a constraint on the energy $\mathcal{E}(\tau) \le \varepsilon x$.*
(i) If $\overline{X}(\tau) \le x + z$ (or equivalently, $\tau \le \sigma(z)$), then a control policy u is optimal if and only if $u(t) = 1$ for $t \in [0, \tau]$ a.e.
(ii) If $\overline{X}(u_{\min}\tau) > z + x$ (or equivalently, $u_{\min}\tau > \sigma(z)$), then there is no feasible control strategy.
(iii) If $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$ (or equivalently, $\tau > \sigma(z) > u_{\min}\tau$), then the best static control policy $u_c(t) = c$ is given by the constant

$$c = \frac{\sigma(z)}{\tau}$$

and the optimal value over the class of static policies is given by

$$J_c^*(z) = \frac{\tau}{\sigma(z)} \int_0^{\sigma(z)} \overline{X}(r)dr.$$

Proof. At the outset we observe that from (9), for *any* control policy, $\overline{X}(u_{\min}t) \le X(t) \le \overline{X}(t)$ for $t \in [0, \tau]$; also $J^*_{u_{\min}}(z) \le J(z) \le J^*_1(z)$. Hence (i) follows since $\overline{X}(\tau) \le x + z$ by assumption, whereas part (ii) is true because $X(\tau) \ge \overline{X}(u_{\min}\tau) > z + x$. Part (iii) follows directly from (3) in view of a change of variable in the expression of $J(z, u)$. $\square$

4.3. Optimal dynamic control

Definition 4.1. A policy u, taking values in $[u_{\min}, 1]$, is called a threshold policy with parameter h if $u(t) = 1$ for $t \le h$ a.e. and $u(t) = u_{\min}$ for $t > h$ a.e.[2]

[2] This is not the most general definition of threshold policy, but one that fits our framework. It is for example also possible for a threshold policy to switch from the lowest value to the highest, but that one will not be relevant for our case. We further note that such policies are also known as *bang–bang* policies with only a single switching time.

In what follows we use the following fact

Lemma 4.1. *Under an optimal policy* $X(\tau) = x + z$.

The proof is deferred to Appendix A.

Theorem 4.2. *Consider the problem of maximizing* $D(\tau)$ *subject to a constraint on the energy* $\mathcal{E}(\tau) \le \varepsilon x$.
(i) *If* $\overline{X}(\tau) \le x + z$ *(or equivalently,* $\tau \le \sigma(z)$*), then a control policy u is optimal if and only if* $u(t) = 1$ *for* $t \in [0, \tau]$ *a.e.*
(ii) *If* $\overline{X}(u_{\min}\tau) > z + x$ *(or equivalently,* $u_{\min}\tau > \sigma(z)$*), then there is no feasible control strategy.*
(iii) *If* $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$ *(or equivalently,* $\tau > \sigma(z) > u_{\min}\tau$*), then an optimal policy is necessarily a threshold one.*

Proof. The proofs of parts (i) and (ii) are the same as those of Theorem 4.1.

We thus proceed with proof of part (iii), working under the assumption $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$.[3] Choose any policy u for which the corresponding trajectory $X(t)$ satisfies $X(\tau) \le z + x$. From Remark 4.1(ii) and (iii) it follows that for any $t \in [0, \tau]$,

$$X(t) \le \min\{\overline{X}(t), \underline{X}(t, z + x)\}. \tag{10}$$

Let

$$h = \sup\{t \in [0, \tau] \mid \overline{X}(t) \le \underline{X}(t, z + x)\}.$$

Define the threshold policy with parameter h:

$$u(t) = \begin{cases} 1 & \text{if } t \le h \\ u_{\min} & \text{if } t > h. \end{cases} \tag{11}$$

The bound (10) is attained with equality when using this policy. Hence it maximizes $\int X(r; u)\mathrm{d}r$ and yet satisfies $X(\tau) = x + z$. $\square$

The way to implement a threshold policy is quite simple. When a mobile has a forwarding opportunity, it checks the age t of the copy of the message, i.e., the time that has elapsed since the message generation time. If t is smaller than the threshold, then the message is forwarded, otherwise it is transmitted with probability $u_{\min}$. Notice that, in principle, global timers are not needed in order to track the age t of a copy of the message. In particular, the age of a copy of the message can be stored in a message field which is zero at generation time and it is increased according to the local clock of forwarding nodes.

4.4. Optimal threshold

Theorem 4.3. *If* $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$ *(or equivalently,* $\tau > \sigma(z) > u_{\min}\tau$*), then the threshold value* h^* *of the optimal policy is given by*

$$h^* = \frac{\sigma(z) - \tau u_{\min}}{1 - u_{\min}}. \tag{12}$$

The optimal value is given by

$$J^*(z) = \begin{cases} \displaystyle\int_0^{\sigma(z)} \overline{X}(r)\mathrm{d}r & \text{if } u_{\min} = 0 \\ \displaystyle\int_0^{h^*} \overline{X}(r)\mathrm{d}r + \frac{1}{u_{\min}} \int_{h^*}^{\sigma(z)} \overline{X}(r)\mathrm{d}r & \text{if } u_{\min} > 0. \end{cases} \tag{13}$$

Proof. The result for the case $u_{\min} = 0$ follows immediately from the definition of $\sigma(z)$. In the case $u_{\min} > 0$, consider any control policy u satisfying $X(\tau) \le x + z$. Then, notice that

$$\sigma(z) \ge s(\tau) = \int_0^{\tau} u(r)\mathrm{d}r$$

where equality holds if and only if $X(\tau) = x + z = \overline{X}(s(\tau)) = \overline{X}(\sigma(z))$.

[3] A different proof, which makes use of the maximum principle, is given in the Appendix.

Since u is positive, s is invertible and we let $t = \eta(s)$. Hence, it follows that

$$J(u) = \int_0^\tau X(t)\mathrm{d}t = \int_0^\tau \overline{X}(s(t))\mathrm{d}t = \int_0^{s(\tau)} \overline{X}(r)\mathrm{d}\eta(r)$$
$$\leq \int_0^\sigma \overline{X}(r)\mathrm{d}\eta(r) = \int_0^\sigma \frac{\overline{X}(r)}{u(\eta(r))}\mathrm{d}r$$

where we used the fact that $\eta'(s) = \frac{1}{u(\eta(s))}$: equality is obtained if and only if $X(\tau) = x + z$.

Among the threshold control policies, the optimal one is thus the one for which h satisfies the constraint $X(\tau) = x + z$, and hence for which h satisfies

$$\sigma(z) = h \cdot 1 + (\tau - h) \cdot u_{\min}$$

which yields (12). The expression for $J^*(z)$ follows accordingly. □

5. Some examples

In this section, we describe how the theory presented specializes in the case of two popular forwarding protocols, namely epidemic routing and two-hop routing introduced in Section 3; we refer the reader to [14] as a standard reference for the use of ODE-based models in DTNs.

5.1. Epidemic routing

Here f is given by

$$f(x) = \rho x(1 - x),$$

where $\rho = N\lambda$. The solution of $\frac{\mathrm{d}x}{\mathrm{d}t} = f(x(t))$ is

$$\overline{X}(t) = \frac{1}{1 + \left(\frac{1}{z} - 1\right)\exp(-\rho t)}.$$

We have

$$\sigma(z) = -\frac{1}{\rho}\log\left(\frac{z(1 - x - z)}{(x + z)(1 - z)}\right)$$

and note that

$$\frac{\mathrm{d}^2\overline{X}(t)}{\mathrm{d}t^2} = \frac{c\rho^2\exp(-\rho t)(c\exp(-\rho t) - 1)}{(1 + c\exp(-\rho t))^3}$$

where $c = 1/z - 1$.

$\frac{\mathrm{d}^2\overline{X}(t)}{\mathrm{d}t^2}$ is seen to be positive for $t \leq S_0$ and negative for $t \geq S_0$, where $S_0 = \log(c)/\rho$. Hence $\overline{X}(t)$ has a sigmoidal form: it is convex for $t \leq S_0$ and concave for $t \geq S_0$.

We also note that

$$\left.\frac{\mathrm{d}\overline{X}(t)}{\mathrm{d}t}\right|_{t=0}^{(epid\text{-}routing)} = z(1 - z)\rho. \tag{14}$$

Using Remark 4.1 we have for a static policy $u(t) = p$:

$$X(t) = \frac{1}{1 + \left(\frac{1}{z} - 1\right)\exp(-\rho p t)}.$$

5.2. Two-hop routing

Here f is given by

$$f(x) = \lambda(1 - x).$$

Then

$$\overline{X}(t) = 1 + (z - 1)\exp(-\lambda t)$$

which is a concave function of t.

Note that

$$\left.\frac{\mathrm{d}\overline{X}(t)}{\mathrm{d}t}\right|_{t=0}^{(two\text{-}hops)} = (1-z)\lambda = z(1-z)\rho/Nz$$
$$= \frac{1}{Nz}\left.\frac{\mathrm{d}\overline{X}(t)}{\mathrm{d}t}\right|_{t=0}^{(epidemic)}. \tag{15}$$

Since $z = \overline{X}(0) \geq 1/N$, we get

$$\left.\frac{\mathrm{d}\overline{X}(t)}{\mathrm{d}t}\right|_{t=0}^{(two\text{-}hops)} \leq \left.\frac{\mathrm{d}\overline{X}(t)}{\mathrm{d}t}\right|_{t=0}^{(epidemic)},$$

with equality for $z = 1/N$.

We have

$$\sigma(z) = -\frac{1}{\lambda}\log\left(\frac{1-x-z}{1-z}\right)$$

and further

$$\bar{J}(z) = \int_0^{\sigma(z)} \overline{X}(r)\mathrm{d}r = \frac{-1}{\lambda}\left[\log\left(\frac{1-x-z}{1-z}\right) + x\right].$$

Optimizing over all static policies

For a static policy $u_c(t) = c$, we have

$$J(z, u_c) = \frac{\bar{J}(z)}{c}$$

which is decreasing in c. The optimal value is given for $c = \frac{\sigma(z)}{\tau}$ which gives

$$J_x^*(z) := \sup_c J(z, u_c) = \tau\left[1 + \frac{x}{\log\left(\frac{1-x-z}{1-z}\right)}\right].$$

6. Extensions on the model

6.1. Undisclosed destinations

We consider now the case when the destination is undisclosed, i.e., the identity of the destination is not known to the source or to the relays. There are some important practical cases when this happens:

- Anonymous destination: the destination of the message is undisclosed in order to preserve anonymity of the recipient
- Broadcasting generates data which are diffused in the network through message relaying; in this case we look at the probability that a tagged node collected the message.

Again, let us assume that at each encounter nodes having a copy of the message forward it with some probability to nodes which do not have the message. We notice that for several forwarding mechanisms, e.g. epidemic routing, the dynamics of the system are still described by the fluid model (2).

We are now interested in the same problem as before: we assume that there exists a finite amount of energy, i.e., a maximum number of copies x of the message, and we want to maximize the probability that the undisclosed destination has the message by time τ.

For undisclosed destinations, the expression for the message delay based on the fluid model is slightly different compared to (4), namely

$$D_u(t) = 1 - (1 - D_u(0))\exp\Big(-N\lambda\int_{s=0}^{t} u(s)X(s)\mathrm{d}s\Big), \tag{16}$$

where the product $u \cdot X$ which appears in (16) is due to the fact that a relay cannot distinguish destinations from other relays. Accordingly, the objective function to be maximized is now

$$J(u) = \int_0^{\tau} u(t)X(t)\mathrm{d}t.$$

Obviously, the cases addressed in Theorem 4.2 still hold. The nontrivial case $u_{\min}\tau < \sigma(z) < \tau$ needs some further derivation.

In the case $u_{\min} > 0$, we can deduce for the cost function

$$J(u) = \int_0^{\tau} u(t)X(t)\mathrm{d}t = \int_0^{\tau} u(t)\overline{X}(s(t))\mathrm{d}t$$
$$= \int_0^{s(\tau)} u(\eta(r))\overline{X}(r)\mathrm{d}\eta(r) = \int_0^{s(\tau)} \overline{X}(r)\mathrm{d}r \leq \int_0^{\sigma(z)} \overline{X}(r)\mathrm{d}r \tag{17}$$

where again we used $\eta'(s) = \frac{1}{u(\eta(s))}$. We also recall that $\sigma(z) \geq s(\tau) = \int_0^{\tau} u(v)\mathrm{d}v$, and equality holds if and only if $\sigma(z) = s(\tau)$. The same reasoning can be applied to the case $u_{\min} = 0$, where $u(t)$ and $X(t)$ are replaced by their restrictions to $I = \{t \in [0, \tau] \mid u(t) \neq 0\}$ in (17).

We note that in the undisclosed destination case the goal is to maximize $X(\tau)$, because the probability to deliver the message to the destination is equal to the number of infected nodes divided by the total number of nodes being that the destination is no more likely to be infected than any other node. This fact is formalized in the following

Theorem 6.1. *Consider the problem of maximizing $D_u(\tau)$ subject to a constraint on the energy $\mathcal{E}(\tau) \leq \varepsilon x$. For undisclosed destinations: cases* (i) *and* (ii) *of Theorem 4.2 still hold;*
(iii) *If $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$ (or equivalently, $\tau > \sigma(z) > u_{\min}\tau$), a policy u is optimal if and only if $X(\tau) = x + z$, and the optimal value is*

$$J^*(z) = \int_0^{\sigma(z)} \overline{X}(s)\mathrm{d}s = \int_z^{x+z} \frac{r}{f(r)}\mathrm{d}r.$$

Also, an optimal control is such that $\sigma(z) = \int_0^{\tau} u(v)\mathrm{d}v$.

Proof. Proof of (iii) follows from the general expression

$$J^*(u) = \int_0^{\tau} u^*(s)X(s)\mathrm{d}s = \int_0^{\tau} \frac{X(s)}{f(X(s))}u^*(s)f(X(s))\mathrm{d}s = \int_z^{x+z} \frac{r}{f(r)}\mathrm{d}r$$

where the last equality holds since the optimal control attains the constraint. □

As we anticipated, for any $u, J(u)$ depends on u only through the value of $X(\tau)$ that u induces. If we denote it by $x(u, \tau)$, then we have

$$J(u) = \int_z^{x(u,\tau)} \frac{r}{f(r)}\mathrm{d}r.$$

This is in contrast to the non-anonymous case where $J(u)$ depended on u through the whole trajectory of x.

Also, the result implies that in the case of undisclosed destinations, there exist several optimal policies. For example, from (17), optimal static control and optimal dynamic control derived before for disclosed destinations are both optimal in the case of undisclosed destinations.

In the case of epidemic routing, for example, the optimal value can be derived as

$$J(u) = \int_z^{x+z} \frac{r}{\rho r(1-r)}\mathrm{d}r = \frac{1}{\rho}\log\frac{1-x}{1-(z+x)}.$$

It is interesting to quantify the penalty in terms of success probability due to the lack of information on the identity of the destination. In particular for static forwarding policies, the optimal value J^* is a factor p^* smaller, which represents exactly the success probability in delivering the message, given that a relay has a copy and meets the destination.

Assume that case (iii) of Theorem 6.1 holds. The difference of the optimal value for undisclosed destination (undisc.) and disclosed destination (disc.) is

$$J^*(z)\Big|^{\text{disc.}} - J^*(z)\Big|^{\text{undisc.}} = \begin{cases} (x+z)(\tau - \sigma(z)) & \text{if } u_{\min} = 0 \\ \dfrac{1-u_{\min}}{u_{\min}} \displaystyle\int_{h^*}^{\sigma(z)} \overline{X}(r)\mathrm{d}r & \text{if } u_{\min} > 0. \end{cases}$$

This can also be explained intuitively as follows. Consider an optimal threshold policy with threshold h^*. In the case of disclosed destination, it is convenient for the relays to infect the largest possible number of nodes as soon as possible, because this increases the probability that one of them will meet the destination before τ. Notice that, in this case, this is also the probability that the message is delivered. In the undisclosed destination case, conversely, infected relays will deliver the message with probability one to the destination up to time h^*. After the threshold expires, however, the probability that they deliver the message when the destination is encountered is $u_{\min}$: thus the advantage of having infected a larger fraction of nodes early is compensated by the fact that infected nodes are forced to forward the message to the destination at lower rates.

Finally, we notice that the two-hop routing algorithm assumes implicitly that the identity of the destination is disclosed, so that the model above does not apply. Nevertheless, it is possible to provide an alternative formulation of the two-hop forwarding algorithm that guarantees the recipient to be undisclosed. In particular, relays do forward the message to any other node which did not receive a copy. But, relays which did not receive the message from the source do not forward it, i.e., every message is still allowed two hops only. We notice that the system dynamics can still be modeled via fluid approximations. However, the application of the original fluid model is not straightforward, because two control variables are involved, namely, the forwarding probability of the source, and the forwarding probability of relays forwarding over the second hop. It is easy to see that, in order to express the delay in closed form, the fraction of nodes infected directly by the source and the fraction of nodes infected by a relay should be accounted separately. The analysis of the resulting system of two differential equations is beyond the scope of the present paper.

6.2. Non-monotone forwarding policies

Consider a monotone policy for which the fluid dynamics is given by (2). Now add to each node a timer; it starts counting whenever the corresponding node receives a copy of the message and expires some exponentially distributed time later. At that time the message is discarded. We also assume that nodes whose timers have expired can be infected again.

The evolution of the state is now modified to

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = u(t)\bar{f}(X(t)) \quad \text{where } \bar{f}(x) = f(x) - \eta x \tag{18}$$

and where $u(t) \in [u_{\min}, 1]$, $u_{\min} \geq 0$.

For the uncontrolled two-hop routing, the corresponding equation is

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = \lambda(1 - X(t)) - \eta X(t),$$

which has the stationary point

$$X(t) = x^* = \frac{\lambda}{\lambda + \eta}.$$

We shall assume throughout that we start initially at $z < x^*$. Then $X(t)$ will converge by monotonically increasing to x^* and will not exceed this value. Thus if we define $y = x/x^*$ to be the normalized fraction of nodes that have a copy of the message, then it indeed satisfies the same constraints $0 \leq y \leq 1$ and $y \geq 0$, and its dynamics is given by

$$\frac{\mathrm{d}Y(t)}{\mathrm{d}t} = u(t)\bar{f}(Y(t))$$

where $\bar{f}(Y(t)) = (\lambda + \eta)(1 - Y(t))$. Thus the normalized state dynamics in the presence of timers is an accelerated version of the one without timers.

Theorem 6.2. *Consider the two-hop forwarding policy with exponential timers with parameter ζ. Consider the relaxed (soft-constrained) problem*

$$\text{Maximize} \quad \int_0^\tau X\mathrm{d}s - \zeta \mathcal{E}(\tau) \tag{19}$$

with $\zeta \geq 0$, and assume that $\varepsilon\zeta < 1$. The optimal policy that solves it also solves the following problem without timers:

$$\text{Maximize} \quad \int_0^\tau X\mathrm{d}s(1 - \epsilon\zeta) - \zeta(X(\tau) - X(0)). \tag{20}$$

Furthermore, the optimal policy is of the threshold type.

Proof. The energy is not any more proportional to $X(\tau) - X(0)$, but instead to $X(\tau) - X(0) + R(\tau)$, where $R(\tau)$ is the total number of deletions of the message due to timers timeout. It is given by $R(\tau) = \zeta \int_0^\tau X(s)\mathrm{d}s$. Thus (19) is equivalent to (20). Now for the last part of the statement of the theorem, note that (19) has an optimal solution as a threshold policy. In fact, consider the solution of (19) and let $x = X(\tau)$ the corresponding terminal value. We observed before that the normalized dynamics is monotone and the constraint on the relevant fraction of messages is x/x^*. Then we can consider the optimal control for the equivalent dynamics X without timers: it meets the constraint, and maximizes the cost. Hence, this policy is optimal under the relaxed formulation (20) as well, and by equivalence the solution to (19) is also of the threshold type. □

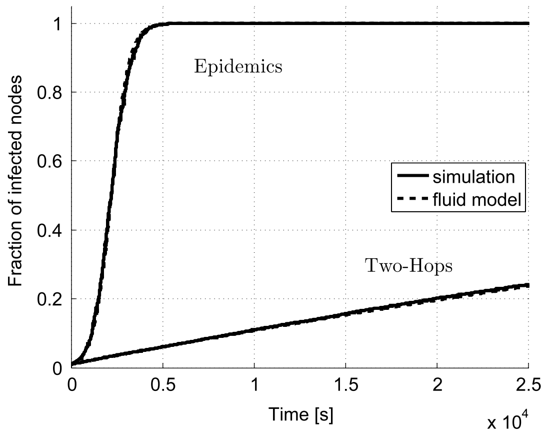

Fig. 1. Fraction of infected nodes, uncontrolled case, $N = 200$.

7. Numerical results

This section complements the previous sections with both simulations and with numerical studies. The objective of the simulations is to check the validity of our modeling and solution approaches: we validate the basic model, study the fluid approximation and get an insight on when it performs well. We then perform a numerical study and compute optimal policies based on the theory that we have developed in previous sections in order to develop insight on the value of static and of dynamic control in DTNs.

The simulation set up is as follows. First, we generated several contact traces based on different mobility patterns, which collect sequences of pairs of nodes coming into radio range, and the time when such contacts occurred (specific settings are reported below). In particular, we used two synthetic mobility models, mobility traces which were generated using Omnet++ according to the Random Waypoint (RWP) mobility model [25], and traces of random contacts occurring according to a marked Poisson process with uniform i.i.d. marks (poissonian traces): the marks distribution is uniform over the set of all (unordered) pairs of nodes and independent conditioned on the Poisson process.

Then, we considered uniform i.i.d. pairs of source–destination nodes, and simulated the (un)controlled forwarding process based on the contact patterns derived from the mobility traces, using Matlab.

7.1. *Uncontrolled forwarding ($u(t) = 1$)*

First, we conducted some experiments aimed at validating the delay formula (4) in the case of uncontrolled forwarding. The first set of numerical examples reported below is referred to as the case where pairs of nodes meet with frequency $\lambda = 1 \times 10^{-5}\ \mathrm{s}^{-1}$; values are averaged over 10^4 samples (confidence intervals were calculated within 95% accuracy).

In both cases of two-hop routing and of epidemic routing, as depicted in Fig. 1, the fluid model follows quite accurately the dynamics of the fraction of infected nodes.

As a further step, we tested the validity of the fluid approximation when changing the number of nodes in the system. In particular, as in Fig. 2, in the case of the epidemic routing, and for the settings considered here, the fraction of infected nodes is approximated accurate by the fluid model as soon as N is on the order of 30 of nodes.

We further validated the model by collecting the statistics for the message delay, namely $D(t)$. As reported in Fig. 3, in particular, the experimental CDF shows a tight match with the theoretical prediction. Notice that, according to Eq. (4), the good fit of the delay CDF confirms the match for the dynamics of $X(t)$.

From these preliminary results we can already characterize some typical features of the two representative routing techniques considered in this paper, namely two-hop routing and epidemic routing. In fact, if we compare Figs. 1 and 3, we notice that in the case of the two-hop relaying protocol, it takes around four times longer than epidemic routing in order to deliver a message with high probability. Of course this was expected, since epidemic routing minimizes the delay at the cost of message overhead. Conversely, the price paid by epidemic routing for such a gain is apparent, since the corresponding fraction of infected nodes, i.e. the energy expenditure, in order to achieve 5 times larger than in the case of two-hop relaying.

Finally, we repeated the experiments on the dynamics of infected nodes and delay using the Omnet++ generated RWP mobility traces. In this setting there are $N = 200$ nodes in the system, which move at $v = 5$ m/s in a square area with side length $L = 5000$ m and have $R = 15$ m radio range. The initial distribution of nodes is drawn from the stationary distribution according to the RWP mobility model [26] in order to avoid transitory effects. Notice that the equivalent value of λ for this case coincides with the value used to generate poissonian contact traces, according to [13]. As a general remark, the numerical results were derived under the conditions $R \ll L$; this choice of the parameters is aimed to mimic conditions for the network to be *sparse*, which is in line with a DTN scenario.

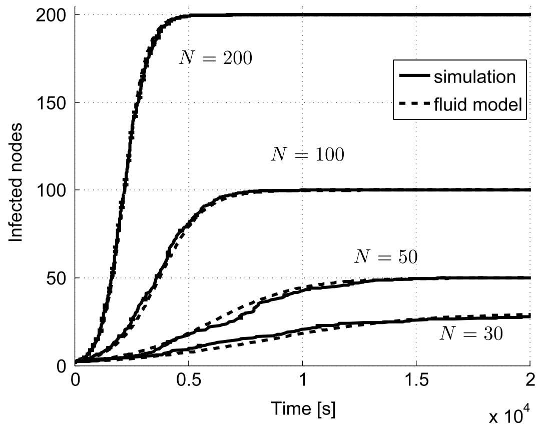

Fig. 2. Fraction of infected nodes, Epidemic Routing, uncontrolled case, various number of nodes $N = 30, 50, 100, 200$.

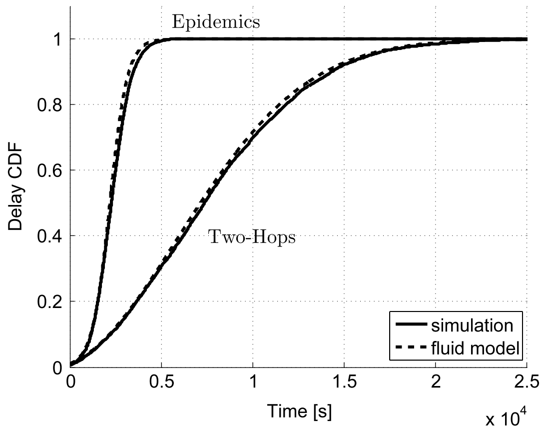

Fig. 3. Delay CDF, uncontrolled case, $N = 200$.

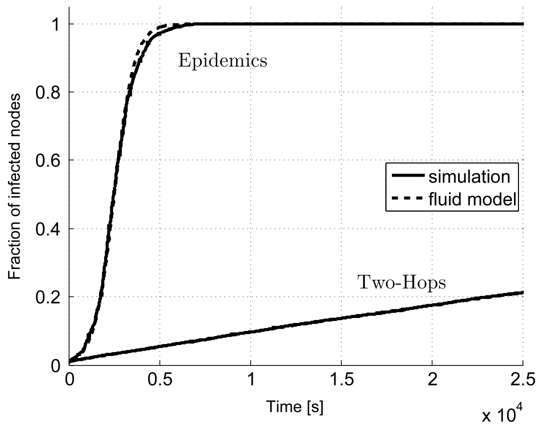

Fig. 4. Fraction of infected nodes, uncontrolled case, RWP mobility, $N = 200$, $L = 5000$ m, $v = 5$ m/s.

As can be observed in Fig. 4 the dynamics of the number of nodes is captured by the model, and even in the case of Fig. 5 the fit of the CDF shows a very tight match with the theoretical prediction.

Also, as seen in Fig. 6, the fluid approximation starts applying when the number of nodes is in the order of some tenth of nodes (30 with the given settings), as already verified in the previous experiments.

In the following we verify the performances of the control policies described previously; the following measurements were obtained using poissonian traces.

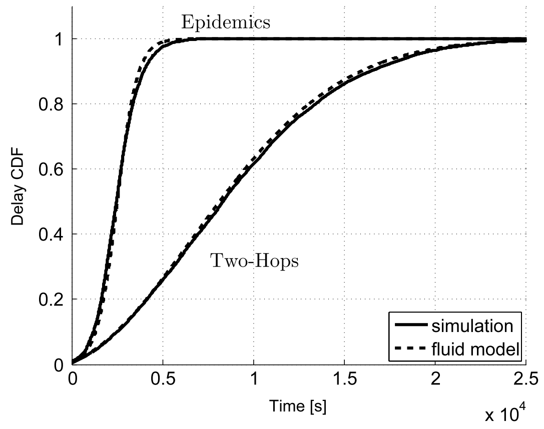

Fig. 5. Delay CDF, uncontrolled case, RWP mobility, $N = 200$, $L = 5000$ m, $v = 5$ m/s.

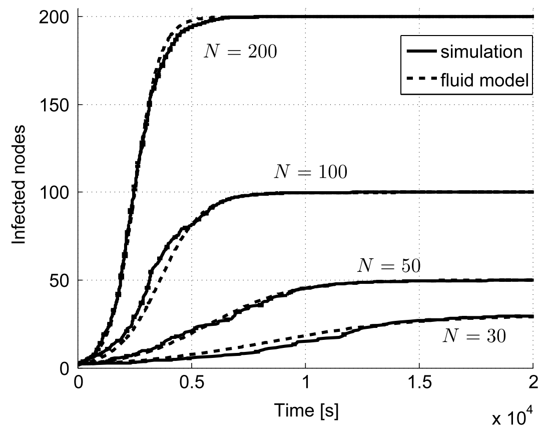

Fig. 6. Fraction of infected nodes, Epidemic Routing, uncontrolled case, RWP mobility, $L = 5000$ m, $v = 5$ m/s, various number of nodes $N = 20$, 50, 100, 200.

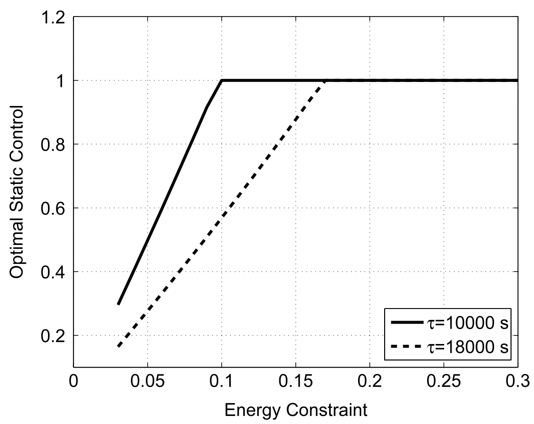

Fig. 7. Optimal static control, $\tau = 10\,000$, $18\,000$ s, two-hop routing.

7.2. *Static control policies* $(u(t) = c)$

After verifying the match of the model with the experimental data, we simulated static control policies under the same settings described before.

In particular, under the two-hop routing policy, we tested two reference values of the maximum permitted delay, $\tau = 10\,000$ and $\tau = 18\,000$, respectively. We then considered an increasing normalized energy constraint and, in particular, the corresponding optimal static control is reported in Fig. 7. The reference performance figure tested is the failure

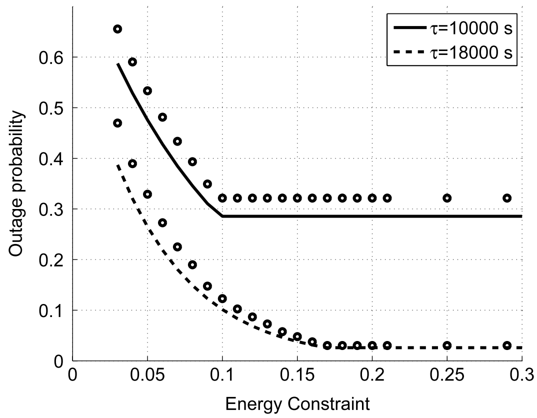

Fig. 8. Failure probability at time $\tau = 10\,000, 18\,000$ when the normalized energy constraint increases, static control case, two-hop routing. Markers indicate simulation outcomes.

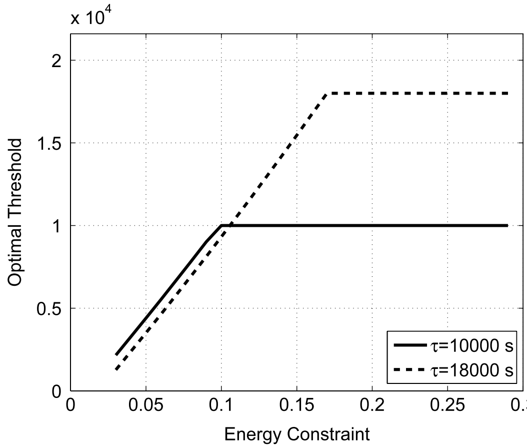

Fig. 9. Optimal threshold, $\tau = 10\,000, 18\,000$ s; two-hop routing.

probability, i.e., $1 - D(\tau)$. As depicted in the figure, the change of slope of the control corresponds to the point beyond which it is optimal to forward messages with unit probability. We also remark that, for the settings considered in this series of experiments, the optimal static control increases almost linearly in the energy constraint.

But, as reported in Fig. 8, while the forwarding probability increases linearly, the failure probability decreases exponentially towards the minimum allowed failure probability, i.e., the value corresponding to uncontrolled forwarding. This suggests that a moderate increase in the forwarding probability has a sensible impact in the delivery probability.

7.3. Dynamic control policies

In the next set of simulations, we verified the performances of the optimal dynamic policies when the normalized energy constraint increases, under the same simulation settings discussed before. We arbitrarily chose $u_{\min} = 0.1$; we remark, though, that according to the optimality of the dynamic policies, within the same feasibility region, the failure probability increases at the increase of $u_{\min}$.

In the case of the two-hop routing policy, as reported in Fig. 9, the optimal threshold h^* increases almost linearly towards the maximum value, i.e., τ, which corresponds to uncontrolled forwarding.

The corresponding behavior of the failure probability is reported in Fig. 10. For the sake of clearness, we also reported the behavior of optimal static policies in the same region: the gain obtained by optimal dynamic policies is quite apparent compared to optimal static control, with a decrease of failure probability of 0.1 below $x = 0.1$.

Incidentally, we notice that in the leftmost regions of Figs. 9 and 10, the graphs do not cover values of the energy constraint such that $x < \tau u_{\min}$; as discussed before, in fact, below such values of the fraction of infected nodes, there exists no feasible forwarding policy.

In order to complete the performance characterization of optimal policies, we depicted the probability distribution function (PDF) of the number of infected nodes under the condition that the message is received by time τ. Basically, this represents the energy expenditure at reception time when $\varepsilon = 1$. As depicted in Fig. 11, when the energy constraint is

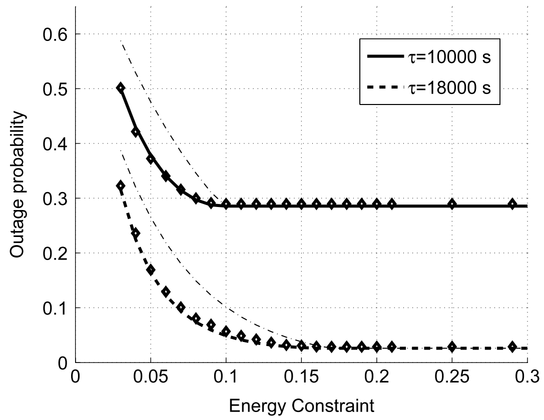

Fig. 10. Failure probability at time $\tau = 10\,000, 18\,000$, when the normalized energy constraint increases; optimal dynamic control, two-hop routing. Markers indicate simulation outcomes, thin dashed lines report optimal static policies for comparison.

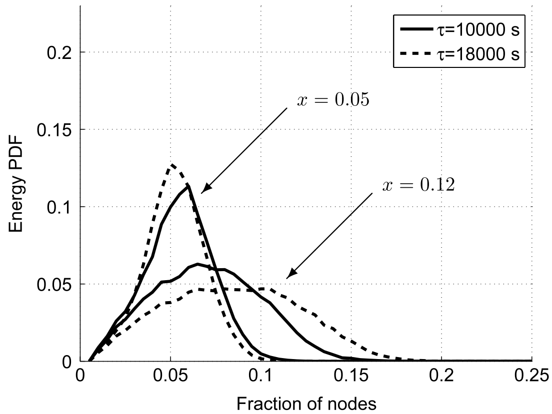

Fig. 11. Conditional PDF of energy expenditure at the reception time $\tau = 10\,000, 18\,000$; energy constraints are $x = 0.05$ and $x = 0.12$, respectively.

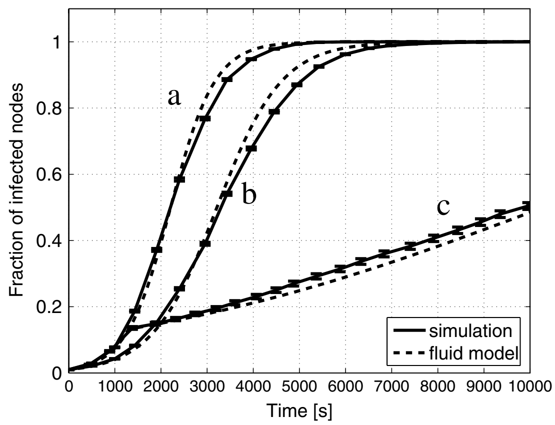

Fig. 12. Dynamics of the fraction of infected nodes in the case of (a) uncontrolled, (b) static and (c) optimal forwarding policies; epidemic routing for $\tau = 2000$ and $x = 0.14$.

tight, the probability distribution function is concentrated around the reference value ($x = 0.05$ in the figure), whereas for a loose bound, practically the uncontrolled case, ($x = 0.12$ in the figure) the PDF is smoother. The shape of the PDF is rather insensitive to the value of τ.

Finally, it is interesting to compare the dynamics of the fraction of infected nodes as reported in Fig. 12 for the case of epidemic forwarding and in Fig. 13 for two-hop routing, under different relaying controls. In particular, with respect to epidemic routing, and with $\tau = 2000$ s and $x = 0.14$, $u_{\min} = 0.1$: with this choice in the case of the static control $u = 0.68$,

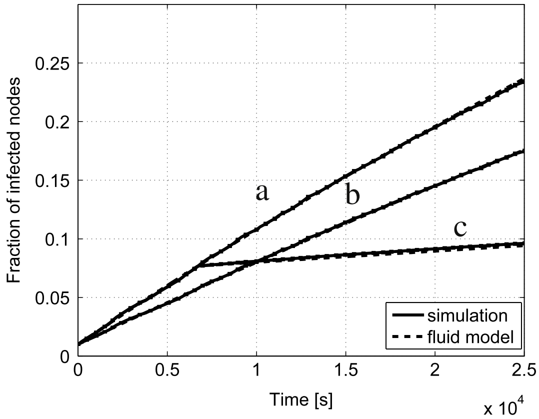

Fig. 13. Dynamics of the fraction of infected nodes in the case of (a) uncontrolled, (b) static and (c) optimal forwarding policies; two-hop routing for $\tau = 10\,000$ and $x = 0.07$.

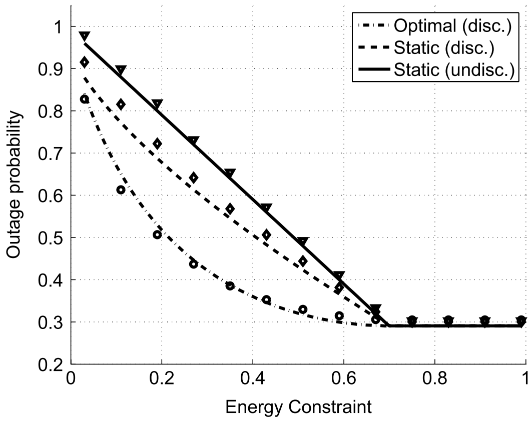

Fig. 14. Failure probability at time $\tau = 3000$ under epidemic routing, comparison of the undisclosed destination scheme (undisc.) and the disclosed destination scheme (disc.); the experiment is repeated for increasing values of the energy constraint; RWP mobility, $N = 200$. Markers indicate simulation outcomes.

whereas in the case of dynamic control, $h^* = 1296$ s. Conversely, in the case of two-hop routing, $\tau = 10\,000$ s, $x = 0.07$, $u_{\min} = 0.1$, and in this case static control $u = 0.7$, whereas in the case of dynamic control, $h^* = 6675$ s.

It is quite apparent that the static control policy generates a delayed version of the uncontrolled dynamics. In the case of the optimal dynamic control, instead, around the threshold, the fraction of infected nodes starts increasing at a slower pace. We notice that the two controlled dynamics, i.e., lines (b) and (c) in Figs. 12 and 13 intersect at time τ: the intersect corresponds to $x + z$ as expected.

7.4. Undisclosed destination

As a final result, we compared the relative performance of the disclosed destination and undisclosed destination forwarding strategies. As reported in Fig. 14, we evaluated the failure probability in the case of epidemic routing for increasing values of the energy constraint. For disclosed destinations, we considered both the case of static policies and optimal policies.

We wish to evaluate relative advantage in performance of the scheme with disclosed destinations. Here, we observe that when the energy constraint is tighter, i.e. for a smaller targeted number of message copies, e.g. $x = 0.3$, the loss in performance in order to preserve anonymity is significant. Compared to the case of disclosed destinations, in particular, we notice that the failure probability is on the order of 0.1 larger in the case of static policies. The loss is even larger when compared to the case of optimal threshold policy with disclosed destination: in this case the difference for the failure probability becomes as large as 0.3. For a larger fraction of targeted infected nodes the difference becomes less significant, as expected.

8. Conclusion

We have studied in this paper the question of how to control efficiently message forwarding in delay tolerant networks. To achieve a desirable tradeoff between a large probability of successful transmission within a given time and the desire to manage resources well (in particular energy), we formulated a constrained optimal control problem based on a fluid model of the system's dynamics. The quantity that we proposed to control was the probability of forwarding a message to another mobile when two mobiles come into each other's transmission range. This control is an additional feature that can be combined with any type of forwarding policy: we have studied in particular its use in conjunction with the two-hop and the epidemic routing policies. We considered both static policies as well as dynamic policies for choosing the message. We identified a threshold structure of the optimal dynamic policies and computed the optimal threshold for the two-hop routing as well as the epidemic routing.

We extended the model to the case of forwarding with undisclosed recipients, i.e., when the identity of the destination is not known. We could characterize some specific performance aspects of such forwarding protocols. A further extension of our model was carried out in the case of non-monotone forwarding policies, where some of the messages are discarded.

Finally, our modeling assumptions and the use of the fluid model were validated through simulations.

Acknowledgments

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Appendix A. Proof of Lemma 4.1

Proof. Let us denote by X^* the state trajectory corresponding to an optimal policy u^* and assume that $X^*(\tau) < x+z$. Let us fix $\delta > 0$ such that $X^*(\tau-\delta) < x+z$ and consider the optimal static policy $u(t) = c^*$ that solves the problem over interval $(\tau-\delta, \tau)$, with initial condition $X^*(\tau-\delta)$. Let X_c be the corresponding state trajectory: by construction, $X_c(\tau) = x+z$. But, due to the continuity of the trajectory, δ can be chosen such that $X_c(t) > X^*(\tau)$ for $\tau-\delta \le t \le \tau$. Then, define the control policy

$$u^\star(t) = \begin{cases} u^* & \text{if } 0 \le t < \tau-\delta \\ c^* & \text{if } \tau-\delta \le t \le \tau. \end{cases}$$

The corresponding state trajectory is $X^\star(t) = X^*(t)$ for $0 \le t \le \tau-\delta$, and $X^\star(t) = X_c(t) > X^*(t)$ for $\tau-\delta \le t \le \tau$. Hence, X^* with $X^*(\tau) < x+z$ cannot be optimal, thus contradicting the starting hypothesis. □

Appendix B. A second proof for Theorem 4.2

Proof. The proofs of parts (i) and (ii) are the same of Theorem 4.1.

We thus proceed with proof of part (iii), working under the assumption $\overline{X}(\tau) > z+x > \overline{X}(u_{\min}\tau)$. We use the Maximum Principle [27]. The Hamiltonian is

$$H(X, u; p) = X - p\,u\,f(X)$$

where p is the co-state variable, which is a continuous, piecewise continuously differentiable function of t. If u is an optimal solution for the original problem, then it maximizes the Hamiltonian, and with the latter being linear in the former, the optimal control takes the two extreme values $u_{\min}$ and 1, depending on whether the product of p and f, $p(t)f(X)$, is positive or negative. But since $f(X)$ is positive for all values of X of interest, we arrive at the simple optimality condition:

$$u(t) = \begin{cases} u_{\min} & \text{if } p(t) > 0 \\ 1 & \text{if } p(t) < 0. \end{cases} \tag{21}$$

Furthermore, the co-state $p(\cdot)$ is obtained as the solution of the adjoint differential equation

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = H_X = 1 - p(t)u(t)f_X(X(t)) \tag{22}$$

where subscript X denotes differentiation with respect to X.

We first make the assumption that $f_X(X) < 0$ for all X in the interval $[z, z+x]$. Now note that since by the condition of this case (iii) neither $u = u_{\min}$ nor $u = 1$ can be optimal policies, we either have the static constant policy derived earlier to be optimal or have a switching policy. The former can immediately be ruled out because p cannot be zero except at isolated points (note that from the co-state equation, if $p(t) = 0$ for some t, then $p(t^+) > 0$, and from the optimality condition (21) u immediately takes one of the extreme values, in this case $u_{\min}$). Hence, optimal u has to take on its extreme values, and since it cannot take on only one of them for the entire interval, as argued earlier, it has to switch between the two. Clearly, which of the two values it takes for a particular t depends solely on the sign of $p(t)$, as dictated by (21). We now consider separately the two possible choices for $p(0)$, positive and negative: If $p(0) > 0$, then it follows from (22) and from the negativity of f_X that $p(t)$ is increasing and hence remains positive. Hence by (21), $u(t) = u_{\min}$ for all t, and this is in contradiction with the hypothesis of case (iii). Thus we have to have $p(0) < 0$, and initially $u = 1$. But we know that this cannot be sustained during the entire interval because it violates the hypothesis of case (iii). Then, at some point u has to switch to $u_{\min}$. But for this switch to take place, we have to have p positive, and once it is positive it remains positive for the remaining time, and hence optimum u does not switch back. Thus there is a threshold h such that $p(t) < 0$ and $u(t) = 1$ for $t < h$ and $p(t) > 0$ and $u(t) = u_{\min}$ for $t > h$.

Now we assume that $f_X(X) > 0$ for all X in the interval $[z, z+x]$. An analysis similar to the one in the previous paragraph leads to the conclusion that it is not possible for p to be positive at $t = 0$, since then it always remains positive (here the argument is somewhat more detailed than the one above, since it is possible for p to decrease initially, but when it hits zero, it immediately starts increasing at a rate 1, and hence never goes negative), and hence $u(t) = u_{\min}$ for all t, a contradiction. If $p(0) < 0$, then initially $u = 1$, and p grows and at some point hits zero and turns positive since the time-derivative of p at zero is positive (which is 1), and hence u switches to its lower value. We know from the previous argument that p cannot become negative after this point, and hence we again have a switching policy with only one switch.

The case when f_X is not sign-definite can be handled along the same lines of the two cases above. We thus conclude that an optimal dynamic policy has to be a threshold one. □

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