Forward Correction and Fountain Codes in Delay-Tolerant Networks

Eitan Altman, *Fellow, IEEE*, and Francesco De Pellegrini, *Member, IEEE*

***Abstract*—Delay-tolerant ad hoc networks leverage the mobility of relay nodes to compensate for lack of permanent connectivity and thus enable communication between nodes that are out of range of each other. To decrease delivery delay, the information to be delivered is replicated in the network. Our objective in this paper is to study a class of replication mechanisms that include coding in order to improve the probability of successful delivery within a given time limit. We propose an analytical approach that allows to quantify tradeoffs between resources and performance measures (energy and delay). We study the effect of coding on the performance of the network while optimizing parameters that govern routing. Our results, based on fluid approximations, are compared to simulations that validate the model.**

***Index Terms*—Delay-tolerant networks (DTNs), forward correction, fountain codes, mobile ad hoc networks.**

I. Introduction

DELAY-TOLERANT ad hoc networks make use of nodes' mobility to compensate for lack of instantaneous connectivity. Information sent by a source to a disconnected destination can be forwarded and relayed by other mobile nodes. There has been a growing interest in such networks as they have the potential of providing many popular distributed services [7], [9], [20].

A naive approach to forward a file to the destination is by epidemic routing in which any mobile device that has the message keeps on relaying it to any other mobile device that falls within its radio range. This would minimize the delivery delay at the cost of inefficient use of network resources (e.g., in terms of the energy used for flooding the network). The need for more efficient use of network resources motivated the use of less costly forwarding schemes such as the two-hop routing protocols. In two-hop routing, the source transmits copies of its message to all mobiles it encounters; relays transmit the message only if they come in contact with the destination. The two-hop protocol was originally introduced in [14].

In this paper, we consider another aspect, i.e., the tradeoff between network resources and delivery probability. In particular, we assume that the message is relevant for a finite amount of time only, i.e., the message is of no use after a certain deadline.

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E. Altman is with INRIA, 06902 Sophia-Antipolis Cedex, France (e-mail: eitan.altman@sophia.inria.fr).

F. De Pellegrini is with Create-Net, 38100 Trento, Italy (e-mail: francesco.depellegrini@create-net.org).

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We assume that the file to be transferred needs to be split into K smaller units. This happens due to the finite duration of contacts between mobile nodes or when the file is large with respect to the buffering capabilities of nodes. Such K smaller units (which we call chunks or frames) need to be forwarded independently of the others. The message is considered to be well received only if all K frames are received at the destination.

After fragmenting the message into smaller frames, it is convenient to better organize the way information is stored in the relay nodes. We aim at improving the efficiency of the delay-tolerant network's (DTN's) operation by letting the source distribute not only the original frames, but also additional redundant frames. This results in a spatial coding of the distributed storage of the frames. We consider coding based on either forward error correction techniques or on network coding approaches. Our main contribution is to provide a closed-form expression for the performance of DTNs (in terms of message delivery probability and energy consumption) as a function of the coding that is used. Also, we derive scaling laws for the success probability of message delivery.

The paper is organized as follows. In Section II, we revisit the state of art and outline the major contributions of the paper. In Section III, we describe the model of the system, and in Section IV we derive the main results for the case of erasure codes. Section V is devoted to the analysis of fountain codes; for both cases, we discover and then study an interesting phenomenon of phase transition. Sections IV and V also involve the design of energy-aware forwarding policies where the source forwards packets with some fixed probability that we optimize. In Section VI, we study an alternative class of forwarding policies, namely threshold-type policies, that achieve the same energy restrictions. The performance figures of the aforementioned coding techniques are then derived for the case of threshold policies. Section VII reports on simulation results in case of synthetic mobility and real-world traces. A conclusion in Section VIII ends this paper.

II. Related Works

Packet-level encoding dates back to the seminal work [1] dealing with packet-level forward error correction (FEC). The author refers to a satellite broadcast protocol where several sites receive a large block of data divided into several frames. Due to channel errors, several sites may need retransmissions, so additional frames are retransmitted. In that context, the problem is to avoid the phenomenon of *ACK implosion* due to several sites requesting repairs. Reference [1] introduces an encoding scheme, based on Reed–Solomon codes. The basic idea is to transmit additional H frames so that, upon receiving any of the $K+H$ total frames, all stations are able to correctly decode the K information frames. The basic advantage there indeed comes from the fact that the redundancy is shared at all broadcast sites. As seen in the following, this scheme and this intuition do prove useful also in our context.

Several works explored the problem of combining FEC techniques and acknowledged retransmission protocols [2]–[4]. The effort there was to meet hard deadlines that represent a strict constraint when multicasting multimedia streams. However, in DTNs the framework is different, and the challenge is to overcome frequent disconnections.

The idea to encode a message using erasure codes and distribute the generated code-blocks over a large number of relays in DTNs has been addressed first in [22] and [15]. The technique is meant to increase the efficiency of DTNs under uncertain mobility patterns. In [22], the performance gain is compared to simple replication, i.e., the technique of releasing additional copies of the same message. The benefit of erasure coding is quantified in that work via extensive simulations for various routing protocols, including two-hop routing. In [15], the case of nonuniform encounter patterns is addressed, showing that there is strong dependence of the optimal successful delivery probability on the allocation of replicas over different paths. The authors evaluate several allocation techniques. Also, the problem is proved to be NP-hard.

General network coding techniques [12] have been proposed for DTNs. In [18], ordinary differential equation (ODE)-based models are proposed for epidemic routing. Semianalytical numerical results are reported describing the effect of finite buffers and contact times; a prioritization algorithm is also proposed.

The work in [23] addresses the use of network coding techniques for stateless routing protocols under intermittent end-to-end connectivity. A forwarding algorithm based on network coding is specified, showing a clear advantage over plain probabilistic routing in the delivery of multiple packets.

Finally, an architecture supporting random linear coding in challenged wireless networks is reported in [10].

1) Novel Contributions: The main contribution of this paper is the closed-form description of the performance of the mean field limit of delay-tolerant ad hoc networks under the two-hop relaying protocol when a message is split into multiple frames. Our fluid model accounts both for the overhead of the forwarding mechanism, captured in the form of a given bound on energy, and the probability of successful delivery of the entire message to the destination within a certain deadline. The effect of coding is included in the model, and both erasure codes and fountain codes are accounted for in closed form. The two coding strategies are characterized in the case of static probabilistic forwarding policies and in the case of threshold policies.

Leveraging the model, the asymptotic properties of the system are derived in the form of scaling laws. In particular, there exists a threshold law ruling the success probability that ties together the main parameters of the system.

III. Model

For the ease of reading, the main symbols used in the paper are reported in Table I.

TABLE I
MAIN NOTATION USED THROUGHOUT THE PAPER

Symbol	*Meaning*
$N-1$	number of relays nodes (excluding the destination)
K	number of frames composing the message
M	number of frames needed to decode with success probability $1-\delta$, $\delta>0$ (fountain codes)
H	number of redundant frames
λ	inter-meeting intensity
τ	timeout value
$X_i(t)$	number of nodes having frame i at time t (excluding the destination)
$X(t)$	sum of the X_is
$\overline{X}(t)$	sum of the X_is when $u_i(t)=1$, $\forall i=1,2,\ldots,K$
$\mathcal{E}(t)$	energy expenditure by the whole network in $[0,\,t]$
x	maximum number of copies due to energy constraint
z	$:=X(0)$
ε	energy per frame
$u_i(t)$	forwarding policy for frame i $u=\sum_i u_i$, $u\in[u_{\min},1]$, $u_{\min}\geq 0$
p_i	static forwarding policy for frame i; $\mathbf{p}=(p_1,p_2,\ldots,p_K)$
p	sum of the p_is
$D_i(\tau)$	probability of successful delivery of frame i by time τ
$P_s(\tau)$	probability of successful delivery of the message by time τ; $P_s(\tau,K,H)$ is used to stress the dependence on K and H

Consider a network that contains $N+1$ mobile nodes: a source, a destination, and $N-1$ relay nodes. We assume that two nodes are able to communicate when they are within reciprocal radio range, and communications are bidirectional. We also assume that contact intervals are sufficient to exchange all frames. This let us consider nodes *meeting times* only, i.e., time instants at which a pair of not-connected nodes fall within reciprocal radio range. Notice that the time needed to exchange a single frame may be much smaller than the time required to exchange the whole file. The problem of optimizing the number of frames generated per message given the statistics of the contact duration is out of the scope of the present work.

Also, let the time between contacts of pairs of nodes be exponentially distributed with given intermeeting intensity λ. The validity of this model been discussed in [13], and its accuracy has been shown for a number of mobility models (Random Walker, Random Direction, Random Waypoint).

We recall that studies based on traces collected from real-life mobility [9] argue that intercontact times may follow a power-law distribution. Recently, the authors of [16] have shown that these traces and many others exhibit exponential tails after a cutoff point. To this respect, the Poisson assumption underlying our model—and thinning arguments that we leverage later on—becomes a tight approximation in the limit when intermeeting intensities are small.

We assume that the transmitted message is relevant during some time τ. We do not assume any feedback that allows the source or other mobiles to know whether the message has made it successfully to the destination within time τ.

The source has a message that contains K frames. If at time t it encounters a mobile that does not have any frame, it gives it frame i with probability u_i, and we let $u=\sum_i u_i\leq 1$ (we shall consider both the case where u_i depends on t and the case where it does not). For the message to be relevant, all K frames should arrive at the destination by time τ. Let $X_i(t)$ be the number of the mobile nodes (excluding the destination) that have at time t a copy of frame i. Denote by $D_i(\tau)$ the probability of a successful delivery of frame i by time τ. Then, given the process X_i (for which a fluid approximation will be used), we have

$$D_i(\tau) = 1 - \exp\left(-\lambda \int_0^\tau X_i(s)ds\right).$$

This expression has been derived in [5] for the fluid model.

The probability of a successful delivery of the message by time τ is thus

$$\begin{aligned} P_s(\tau) &= \prod_{i=1}^{K} D_i(\tau) \\ &= \prod_{i=1}^{K} \left[1 - \exp\left(-\lambda \int_0^\tau X_i(s)ds\right)\right] \end{aligned}$$

where we assumed that the success probability of a given frame is independent of the success probability of other frames. This decoupling assumption is confirmed by our numerical experiments. This decoupling argument holds in the mean field limit as shown later.

A. Fluid Approximations

In the following, we shall often write λ^N instead of λ in order to allow later for a scaling in N.

Let $X(t) = \sum_{i=1}^K X_i(t)$. Then, we introduce the following standard fluid approximation [24] based on mean field analysis:

$$\frac{dX_i(t)}{dt} = u_i(t)\lambda^N (N - X(t)). \qquad (1)$$

Taking the sum over all i, we obtain the separable differential equation

$$\frac{dX(t)}{dt} = u(t)\lambda^N (N - X(t)) \qquad (2)$$

whose solution is

$$X(t) = N + (z - N)e^{-\lambda^N \int_0^t u(v)dv}, \quad X(0) = z.$$

Thus, $X_i(t)$ is given by the solution of

$$\frac{dX_i(t)}{dt} = -u_i(t)\lambda^N (z - N)e^{-\lambda^N \int_0^t u(v)dv}. \qquad (3)$$

To prove the convergence of the approximation, it will be convenient to define $\bar{\lambda} := N\lambda^N$ and $\hat{X}_i := X_i/N$. We then have

$$\frac{d\hat{X}_i(t)}{dt} = u_i(t)\bar{\lambda}(1 - \hat{X}(t)).$$

The reason we start with the definition of $X_i(t)$ rather than work directly with $\hat{X}_i$ is that it has a simple interpretation as the expectation of X_i!.

We notice that the mean field approximation (1) includes a multiplicative factor $u_i(t)$, which follows from the probabilistic control operated at the source. Since the source sends a frame to a relay node with no frames with probability $u_i(t)$, independent of the intermeeting process, the forwarding process is equivalent to the *thinning* of the intermeeting process, which is Poisson. Hence, the forwarding process also is a Poisson process. In Section III-B, we show that these fluid models are a valid approximation for the controlled forwarding schemes that we consider in this paper.

B. Mean Field Analysis

Let N be the total number of nodes, excluding the destination. Denote the rate of intermeeting times between the source and any other node by λ^N. Let $x_i^N(t)$ be the number of mobiles with a copy of frame i at time t (excluding the destination). This is a continuous-time controlled Markov chain. Let $\mathbf{x}$ stand for the vector of the x_i's, and let $\mathbf{u}$ stand for the vector of the u_i's. Let x stand for the sum of the components of $\mathbf{x}$. Let $\mathbf{e}_i$ be the vector whose ith entry is 1, and all other entries are zero. The rates of transitions are given by

$$\rho(\mathbf{x}^N(t), \mathbf{x}^N(t) + \mathbf{e}_i, \mathbf{u}_t) = u_i(t)\lambda^N (N - x^N(t)).$$

Assume that the limit $\bar{\lambda} = \lim_{N\to\infty} N\lambda^N$ exists. Let $\mathbf{x}^N(0)/N = \hat{\mathbf{X}}(0)$. Assume that $u_i(t)$ are Lipschitz-continuous. Then, Kurtz's Theorem [25, Theorem 5.3] implies that

$$P\left(\sup_{0\le t\le T} \|\frac{\mathbf{x}^N(t)}{N} - \hat{\mathbf{X}}(\mathbf{t})\| > \varepsilon \Big| \mathbf{x}^N(0) = N\hat{\mathbf{X}}(\mathbf{0})\right) \le C_1 e^{-NC_2(\varepsilon)} \qquad (4)$$

where $C_1 > 0$ and C_2 is positive and quadratic near $\varepsilon = 0$. This implies that $\hat{\mathbf{X}}(t)$ is the limit of $\mathbf{x}^N$ almost surely and in probability over the interval $[0, T]$.

Now, the number of contacts of the destination with each mobile during the interval $[0, T]$ is a Poisson random variable with parameter λ^N. Thus, conditioned on the process $\mathbf{x}^N$, the number of contacts between the destination and a node having frame i is Poisson with parameter $\Lambda_i(x_i^N) = \lambda^N \int_0^\tau x_i^N(s)ds$. Thus, the probability that there is no contact between the destination and a node having a frame i during $[0, T]$ is $\exp(-\Lambda_i)$. Now Λ_i, $i = 1, \ldots, K$, are independent (conditioned on $\mathbf{x}^N$). Hence, the probability of successful delivery in the finite population model is given by

$$P_s^N(T) = E\prod_{i=1}^{K}\Big(1 - \exp(-\Lambda_i(x_i^N))\Big).$$

Since we showed that the stochastic process $\mathbf{x}^N(t)$ converges to $\hat{\mathbf{X}}(t)$ in probability (in the sup-norm), the expectation of any continuous function of the process $\mathbf{x}^N(t)$ converges to the function of the limit of the process. This implies that $P_s^N(T)$ converges to P_s^T.

Remark 3.1: In the rest of the paper, we shall use two main types of policies: constant policies (clearly Lipschitz-continuous) and threshold policies. A threshold policy is not Lipschitz-continuous at one point in time, but it is constant elsewhere. Thus, one can apply the above theorem separately to the convergence before and after that instant. This argument can be used to show optimality of the threshold policies within a class of policies that are piecewise Lipschitz-continuous (i.e., with finitely many points in which the function is not Lipschitz-continuous).

1) Constant Policies: In the case of constant policies, we let $u_i(t) = p_i$, $\mathbf{p} := (p_1, p_2, \ldots, p_K)$, and $p = \sum_i p_i$. Hence, it follows

$$X_i(t) = X_i(0) + (N - z)\frac{p_i}{p}\left[1 - e^{-\lambda p t}\right]. \quad (5)$$

Let us assume $X_i(0) = 0$ for $\forall i = 1, 2, \ldots, K$. Hence

$$X_i(t) = N\frac{p_i}{p}\Big(1 - e^{-\lambda p t}\Big). \quad (6)$$

C. Taking Erasures Into Account

So far, we have assumed that the transmission of a frame is always successful. Assume that this is not the case and that the transmission of a frame fails with some probability q. We assume that the process describing whether packet transmissions are successful or not is i.i.d. We assume moreover that a packet that suffers from unrecoverable transmission errors at a mobile is discarded so that it does not occupy memory space in the relay node; this ensures that such a mobile node can still act as a relay at the next meeting with another mobile having a packet to be transmitted.

Losses such as those just described do not need an extra modeling: We may replace the rate λ of intermeetings between two nodes by $\lambda(1-q)$, i.e., the rate of the potentially successful intermeetings of the nodes. This can be used in the equations that we derived in describing the dynamics of the system and its performance measures.

An additional type of loss may occur at the destination. This is the case when it is not mobile and it is connected to an external network (possibly a wired one). In this case, losses may occur in that part of the network; this is the case for example when several stations may compete to transmit to the same gateway destination so that losses are due to the combined effect of finite contact durations and to contention among multiple relays.

Assume that a loss there occurs with probability q'. We do not assume any feedback that would allow the DTN to know about events that occur at the external network. In order to be able to recover from such losses, we assume that the destination may keep receiving copies of the same frame. In particular, a mobile that has transmitted a frame to the destination will keep the copy and could try to retransmit it to the destination at future intermeeting occasions. This additional type of loss process does not alter the fluid dynamics of $X_i(t)$. Its impact on the performance is by replacing λ in (1) by $\lambda(1-q')$.

D. Nonconstrained Problem

The success probability when using $\mathbf{p}$ is

$$\begin{aligned} P_s(\tau, \mathbf{p}) &= \prod_{i=1}^{K}\left(1 - \exp\left(-\lambda\int_0^{\tau} X_i(v)dv\right)\right) \\ &= \prod_{i=1}^{K}\left[1 - \exp\left(-\frac{\lambda}{p}\int_0^{\tau} N p_i\Big(1 - e^{-\lambda p v}\Big)dv\right)\right] \\ &= \prod_{i=1}^{K} Z(p_i) \end{aligned} \quad (7)$$

where

$$\begin{aligned} Z(p_i) &:= 1 - \exp\Big(L(\tau, p)p_i\Big) \\ L(\tau, p) &:= \frac{N}{p^2}\Big(1 - \lambda p\tau - e^{-\lambda p \tau}\Big). \end{aligned} \quad (8)$$

For fixed ratios p_i/p, $P_s(\tau, \mathbf{p})$ is increasing in p and is maximized at $p = 1$.

Let $P_s^*(\tau)$ be the optimal delivery probability for the problem of maximizing $P_s(\tau)$ with p fixed. The proof of the following can be found in the Appendix.

Theorem 3.1: $\mathbf{p}^* = (p/K, \ldots, p/K)$ is the unique solution to the problem of maximizing $P_s(\tau)$ s.t. $\sum_i p_i = p$, $p_i \geq 0$.

E. Constrained Problem

Denote by $\mathcal{E}(t)$ the energy consumed by the whole network for the transmission of the message during the time interval $[0, t]$. It is proportional to $X(t) - X(0)$ since we assume that the message is transmitted only to mobiles that do not have the message, and thus the number of transmissions of the message during $[0, t]$ plus the number of mobiles that had it at time zero is equal to the number of mobiles that have it. Also, let $\varepsilon > 0$ be the energy spent to forward a frame during a contact. We thus have $\mathcal{E}(t) = \varepsilon(X(t) - X(0))$. Notice that ε represents the energy to transmit and receive a copy of the message. It should also be noted that we neglected here the term related to the idle-mode power consumption. In fact, it can be viewed as a linear term in τ, and as such it is not accounted for in our optimization framework.

In the following, we will denote the maximum number of copies that can be released due to the energy constraint as x.

We compute in particular the optimal probability of successful delivery of the message by some time τ under the constraint that the energy consumption till time τ is bounded by some positive constant.

Define $\bar{X}(t)$ to be the solution of (2) when $u(t) = 1$, i.e.,

$$\bar{X}(t) = N + (z - N)e^{-\lambda t}.$$

Note that $X(t) = \bar{X}(pt)$.

Denote $\sigma(z) := \bar{X}^{-1}(x + z)$ given $\bar{X}(0) = z$, which is the time elapsed until x extra nodes (in addition to the initial z ones) receive the message in the uncontrolled system. We have

$$\sigma(z) = -\frac{1}{\lambda}\log\left(\frac{N - x - z}{N - z}\right). \quad (9)$$

For $p = \sigma(z)/\tau$, we obtain the expression

$$L(\tau, p) = \frac{N}{p^2}\left(\log\left(1 - \frac{x}{N - z}\right) + \frac{x}{N - z}\right). \quad (10)$$

For the sake of generality, let us assume that the forwarding control $u_{\min} \leq u \leq 1$, $u_{\min} \geq 0$.

Theorem 3.2: Consider the problem of maximizing $P_s(\tau)$ subject to a constraint on the energy $\mathcal{E}(\tau) \leq \varepsilon x$.

(i) If $\bar{X}(\tau) \leq x + z$ (or equivalently, $\tau \leq \sigma(z)$), then a control policy u is optimal if and only if $p_i = 1/K$ for all i.

(ii) If $\bar{X}(u_{\min}\tau) > z + x$ (or equivalently, $u_{\min}\tau > \sigma(z)$), then there is no feasible control strategy.

(iii) If $\bar{X}(\tau) > z + x > \bar{X}(u_{\min}\tau)$ (or equivalently, $\tau > \sigma(z) > u_{\min}\tau$), then the best control policy is given by $p_i = p^*/K$ where

$$p^* = \frac{\sigma(z)}{\tau} \tag{11}$$

and the optimal value is

$$P_s^*(\tau) = \left[1 - \left(1 - \frac{x}{N-z}\right)^{\frac{N}{p^*K}} \exp\left(-\frac{N}{p^*K}\frac{x}{N-z}\right)\right]^K.$$

Furthermore, if $u_{\min} = 0$, an optimal control policy is always feasible as given either by (i) or (iii).

Proof: Parts (i) and (ii) are obvious. Part (iii) follows from the fact that $X\left(\frac{\sigma(z)}{\tau}\tau\right) = \bar{X}(\sigma(z)) = x+z$, so the energy bound is attained for $p^* = \frac{\sigma(z)}{\tau}$. Also, the expression for $P_s^*(\tau)$ follows from an immediate application of the result in Theorem 3.1 to (8). ■

Remark 3.2: Recall that we base our analysis on a fluid model. Thus, for a finite number of nodes, the optimality of the control $p^* = \frac{\sigma(z)}{\tau}$ is meant to hold on average. Now, let us fix a target number of infected nodes $\tilde{x}$. In the mean field approximation, $X(\tau)$ converges almost surely to $\tilde{x}$ for $N \to \infty$.

IV. Adding Fixed Amount of Redundancy

Consider now the case when some extra frames are sent by the source when using erasure codes. The fluid model that describes the relaying process is obviously not changed, so we can simply add H redundant frames and consider the new message that now contains $K + H$ frames

Again, if at time t the source encounters a mobile that does not have any frame, it gives it frame i with probability p.

Let $S_{n,p}$ be a binomially distributed r.v. with parameters n and p, i.e.,

$$P(S_{n,p} = m) = B(p, n, m) := \binom{n}{m} p^m (1-p)^{n-m}.$$

The probability of a successful delivery of the message by time τ is thus

$$P_s(\tau, K, H) = \sum_{j=K}^{K+H} B(D_i(\tau), K + H, j) \tag{12}$$

where $D_i(\tau) = 1 - \exp(-\lambda \int_0^\tau X_i(s)ds)$.

A. Main Result

Following the intuition, one would expect that the increase of redundancy is beneficial and the probability of success should increase with H. The proof of this fact makes use of concavity arguments, and it is resumed in the following.

Lemma 4.1: The maximum of $P_s(\tau, K, H)$ over $\{p_m \geq 0,\ m = 1, \ldots, K + H\}$ under the constraint $\sum_{i=1}^{K+H} p_i = p$ is achieved at $p_m = p/(K + H)$.

Proof: Let $A(K, H)$ be the set of subsets $h \subset \{1, \ldots, K + H\}$ that contain at least K elements. For example, $\{1, 2, \ldots, K\} \in A(K, H)$. Fix p_i such that $\sum_{i=1}^{K+H} p_i = p$. Then, the probability of successful delivery by time τ is given by

$$P_s(\tau, K, H) = \sum_{h \in A(H,K)} \prod_{i \in h} Z(p_i).$$

For any i and j in $\{1, \ldots, K + H\}$, we can write

$$P_s(\tau, K, H) = Z(p_i)Z(p_j)g_1 + (Z(p_1) + Z(p_2))g_2 + g_3$$

where g_1, g_2, and g_3 are nonnegative functions of $\{Z(p_m),\ m \neq i,\ m \neq j\}$. For example

$$g_1 = \sum_{h \in A_{\{i,j\}}(H,K)} \prod_{\substack{m \in h \\ m \neq i,\ m \neq j}} Z(p_m)$$

where $A_v(K, H)$ is the set of subsets $h \subset \{1, \ldots, K + H\}$ that contain at least K elements and such that $v \subset h$.

Now, consider maximizing $P_s(\tau, K, H)$ over p_i and p_j that satisfy $p' = p_i + p_j$ for a given $p' \leq p$. Since $Z(\cdot)$ is strictly concave, it follows by Jensen's inequality that $Z(p_i) + Z(p_j)$ has a unique maximum at $p_i = p_j = p'/2$. This is also the unique maximum of the product $Z(p_i)Z(p_j)$ (using the same argument as in (16)) and hence of $P_s(\tau, K, H)$.

Since this holds for any i and j and for any $p' \leq p$, this implies the lemma. ■

Using the same arguments as those that led to Theorem 3.2, together with Lemma 4.1, yields the following.

Theorem 4.1: Consider the problem of maximizing $P_s(\tau, K, H)$ over p_i, $i = 1, \ldots, K + H$, subject to a constraint on the energy $\mathcal{E}(\tau) \leq \varepsilon x$.

(i) If $\bar{X}(\tau) \leq x + z$ (or equivalently, $\tau \leq \sigma(z)$), then a control policy u is optimal if and only if $p_i = 1/(K + H)$ for all i.

(ii) If $\bar{X}(u_{\min}\tau) > z + x$ (or equivalently, $u_{\min}\tau > \sigma(z)$), then there is no feasible control strategy.

(iii) If $\bar{X}(\tau) > z + x > \bar{X}(u_{\min}\tau)$ (or equivalently, $\tau > \sigma(z) > u_{\min}\tau$), then the best control policy is given by $p_i = p^*/(K + H)$ where p^* is given in (11) and the optimal value is

$$P_s^*(\tau, K, H) = \sum_{j=K}^{K+H} B(\hat{p}, K + H, j)$$

where

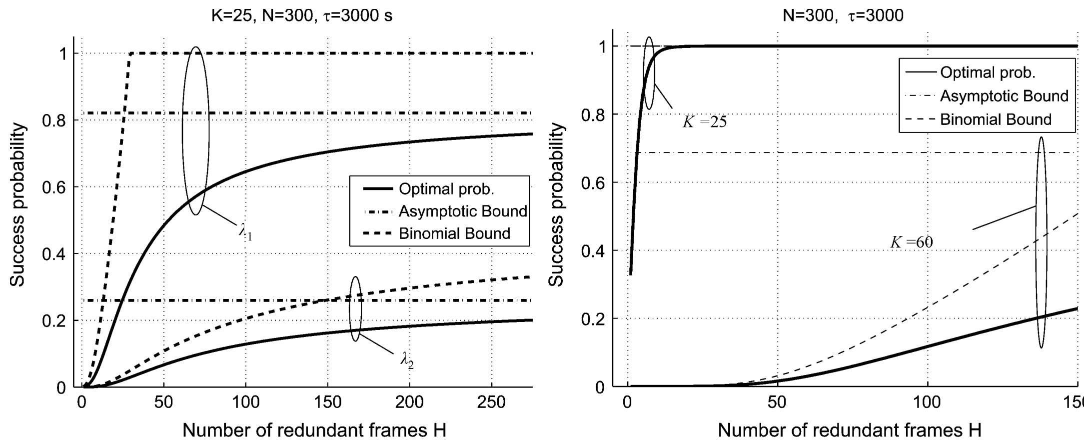
$$\hat{p} = 1 - \exp\left(L(p^*, \tau)p^*/(K + H)\right).$$

Furthermore, if $u_{\min} = 0$, an optimal control policy is always feasible as given either by (i) or (iii).

B. Properties and Approximations

We now derive further characterizations for the optimal success probability. These results will provide both bounds for the case of block coding and help also in the analysis of fountain codes.

Corollary 4.1: $P_s^*(\tau, K, H)$ is increasing with H.



(a) (b)

Fig. 1. (a) Success probability for $N = 300, K = 25, p = 1$ under different values of λ; $\lambda_1 = 0.091 \cdot 10^{-03}$ and $\lambda_2 = 0.065 \cdot 10^{-03}$. (b) Success probability for $\lambda = 0.22 \cdot 10^{-03}$, $N = 300, p = 1$ under different values of $K = 35, 60$.

Proof: We make the following observation. Fix K and H and a vector $(p_1, \ldots, p_{K+H})$ whose entries sum up to p^* [given in (11)]. Then, the success probability is the same as when increasing the redundancy to $H+1$ and using the vector $(p_1, \ldots, p_{K+H}, 0)$. By definition, the latter is strictly smaller than $P_s^*(\tau, K, H+1)$ (which is the optimal success probability with $H+1$ redundant packets).

The above is in particular true when taking $p_i = p^*/(K+H)$ (which maximizes $P_s(\tau, K, H)$), and hence $P_s^*(\tau, K, H) < P_s^*(\tau, K, H+1)$. ■

Now, we provide two useful bounds for $P_s^*(\tau, H, K)$.

First, it is possible to derive the asymptotic approximation of $P_s^*(\tau, H, K)$ for $H \longrightarrow \infty$. For the sake of notation, let $V = L(p, \tau)p$. The following holds:

$$\begin{aligned} P_s^*(\tau, H, K) &= e^V \sum_{s=K}^{H+K} \binom{H+K}{s} \left(e^{-\frac{V}{H+K}} - 1\right)^s \\ &= 1 - e^V \sum_{s=0}^{K-1} \binom{H+K}{s} \\ &\quad \times \left(\frac{(-V)^s}{(H+K)^s} + o((H+K)^{-s}) \right). \end{aligned}$$

For large values of $H+K$, the sth term of the right-hand summation writes

$$\begin{aligned} \binom{H+K}{s} \left(\frac{-V}{H+K} \right)^s &= \frac{(H+K)!}{s!(H+K-s)!} \frac{(-V)^s}{(H+K)^s} \\ &\sim \frac{(-V)^s}{s! e^s} \frac{1}{\left(1 - \frac{s}{H+K}\right)^{H+K}} \\ &\sim \frac{(-V)^s}{s!} \end{aligned}$$

where the Stirling formula applies, $n! \sim \sqrt{2\pi n}\,(\frac{n}{e})^n$, with $f \sim g$ meaning $\lim_{H \longrightarrow \infty} f/g = 1$.

Corollary 4.2: For $K \geq 1$ and $\tau \geq 0$

$$\lim_{H \longrightarrow \infty} P_s^*(\tau, H, K) = 1 - e^{L(p,\tau)p} \sum_{s=0}^{K-1} \frac{[-L(p, \tau))p]^s}{s!}.$$

We note that in sight of the monotonicity stated in Corollary 4.1, the limit is reached from below.

Also, the binomial bound [6, Theorem 1.1] applies. For $1 \leq m \leq n-1$, define d through $m = \lceil dpn \rceil$. Then

$$\begin{aligned} &P(S_{n,p} \geq m) \\ &\quad \leq \frac{n^{1/2}}{(2\pi m(n-m))^{1/2}} \frac{d^{1-dpn}}{1-d} \left(\frac{1-p}{1-dp} \right)^{(1-dp)n}. \end{aligned}$$

In Fig. 1(a), we reported the numerical comparison of the optimal delivery probability as a function of the number of redundant frames for a particular setting. The asymptotic bound is reported as an horizontal dot-slashed line; the binomial bound is reported with a slashed line. We notice that the binomial bound provides a good approximation for lower success probabilities, i.e., at smaller values of λ. The relative increase of the success probability under erasure codes is apparent. For instance, the success probability with $\lambda = 0.22 \cdot 10^{-03}$ increases from 0.12 for $H = 0$ to one with a few redundant frames ($H = 12$). The maximum attainable improvement, though, is dictated by the upper bound.

In Fig. 1(b), the optimal delivery probability is depicted as a function of the number of redundant frames, at the increase of K. Even in this case, the binomial bound proves a better approximation for lower success probabilities, as it appears in the case of $K = 60$.

C. Phase Transition

In what follows, we elaborate based on results from Theorem 4.1, and we study the case of large values of N. Let us assume that the total number of frames, $H+K$, grows as a function of N, i.e., $H+K = (H+K)(N)$.

The question we would like to answer is, in the asymptotic regime, what is the effect of redundancy onto the delivery probability—that is, how the number of redundant frames should scale with respect to the total number of frames.

We assume throughout that the following limits exist:

$$\hat{K} := \lim_{N \to \infty} K(N)/N \quad \text{and} \quad \hat{H} := \lim_{N \to \infty} H(N)/N.$$

Notice that this implies, of course, that the energy constraint should grow at most linearly with N, and we thus assume that the limit $\hat{x} := \lim_{N \to \infty} x(N)/N > 0$ exists.[1]

Theorem 4.2: Introduce the threshold

$$\Gamma_0 := \lambda\tau \left(1 + \frac{\hat{x}}{\log(1-\hat{x})}\right)$$

then the following holds:

$$\lim_{N \to \infty} P_s(\tau, K, H) = \begin{cases} 0, & \text{if } \hat{K} + \hat{H} \le \Gamma_0 \\ 0, & \text{if } \hat{K} + \hat{H} > \Gamma_0 \text{ and } \hat{K} > \Gamma_0 \\ 1, & \text{if } \hat{K} + \hat{H} > \Gamma_0 \text{ and } \hat{K} < \Gamma_0 \end{cases}.$$

The proof is in the Appendix. We conclude that there exists a phase-transition effect. Its threshold Γ_0 is the same as that we shall obtain later for the fountain codes.

Remark 4.1: The way we interpret the result in Theorem 4.2 is that, in order to deliver the message with high probability, for large N, the number K of information frames of a message should not exceed the threshold Γ_0, but the sum of the encoded ones should. We comment further on these aspects in Section VIII.

D. Scalability

When considering erasure codes to perform packet-level FEC, a typical concern is the complexity of coding/decoding procedures. Notice, however, that in our framework, encoding is operated at the source and decoding at the destination. For a large number of nodes, we would like to relate the complexity of the coding scheme to the network parameters; the result derived in Theorem 4.2 provides us the connection. In particular, consider a block code $C = (n, k, d)$, where n is the length of the code and d its distance. Recall that any error-correcting code with distance d is able to correct up to τ errors and e erasures as long as $2\tau + e \le t = \lfloor (d-1)/2 \rfloor$ [27].

In what follows, assume the code to be linear. Hence, $|C| = q^k$, and $C \subset (F_q)^n$ for some finite field $\mathbb{F}_q$.

In particular, denote $n = (\hat{K} + \hat{H})N$ and $k = \hat{K}N$, and assume for simplicity that we choose a sequence of codes with increasing N in order to satisfy the following relations to hold at least for any $N > N_0$:

$$n > \Gamma_0 N \quad k < \Gamma_0 N$$

so that $P_s(\tau, K, H) \ge 1 - \gamma_0$ for some $\gamma_0 > 0$ according to Theorem 4.2. Practical reasons suggest to use linear systematic codes for which efficient decoding procedures exist. The customary choice for erasure codes suggested in literature is provided by Reed–Solomon (RS) codes, which are MDS codes for which it holds $d = n - k + 1$ and $n = q - 1$ [27]. Notice that the MDS assumption that has been assumed before ensures the linearity of the number of recovered frames in the overhead introduced by the coding scheme, i.e., $h = n - k$; this is not the case for non-MDS codes, e.g., the fountain codes that we will describe in Section V.

Thus, we see that the above relation for such codes writes

$$q > \lambda\tau N\left(1 + \frac{\hat{x}}{\log(1-\hat{x})}\right) + 1.$$

In particular, coding operations for RS codes have complexity $O(q \log q)$ [28].

Hence, the above relation bounds the complexity of the decoding scheme by tying together the physical parameters of the system—namely the intermeeting intensity, the deadline, and the energy constraint. We observe, in particular, that the stricter the constraint on the energy, the larger q, i.e., the complexity of decoding. Notice that the above description can be easily extended to the broader class of linear MDS codes for which the MDS conjecture (verified for all practical cases of interest) suggests that $n \le q + 2$ [27]. With the same notation used before, here we can write

$$q > \lambda\tau N\left(1 + \frac{\hat{x}}{\log(1-\hat{x})}\right) - 2.$$

For general MDS codes, however, coding operations may have much higher complexity, i.e., $O(q^3)$. Notice that so far we have been dealing with codes asymptotically showing a positive code rate (actually $1/(1 + h/k)$ with the above notation). A more efficient technique for the encoding of information frames is described in the following in the case of rateless codes that include fountain codes as a special case.

V. Fountain Codes

We have recalled in Section IV that decoding erasures is easier than decoding errors. This is due to the fact that the locations of errors are known. For such a reason, there exist very efficient techniques for decoding erasures based on (randomized) low-density parity check codes. Fountain codes [28], in particular, are binary codes and can be encoded and decoded with linear complexity, namely $O(n)$.

Each time the source meets a node, it sends to it (with probability p) a packet obtained by generating a new random linear combination of the K original packets. Using fountain codes, we know that for any δ, in order for the destination to be able to decode the original message with probability at least $1 - \delta$, it has to receive at least $M := K \log(K/\delta)$ packets [19, Ch. 50]. A useful expression for large K will be used later. If we write $M = K(1 + \alpha)$, then α that guarantees the destination can decode the original message with probability of at least $1 - \delta$ is given by

$$\alpha = \frac{(\log(K/\delta))^2}{\sqrt{K}}.$$

[1]In what follows, we will exclude the trivial case $\hat{x} = 0$, which corresponds to the case when no relaying is allowed.

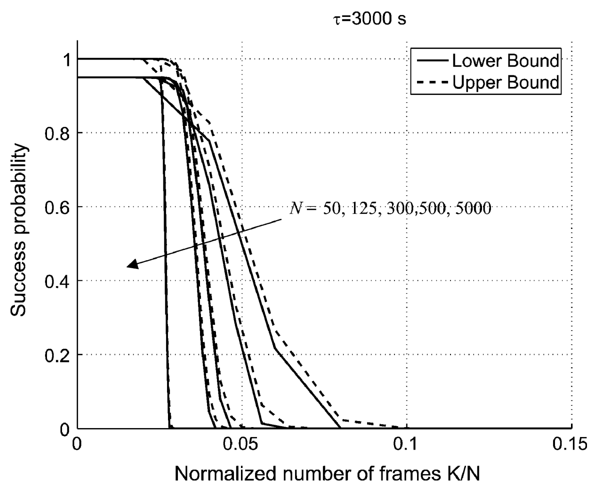


Fig. 2. Upper and lower bound $p=1, \delta=0.02$ as given in Corollary 5.1, for various values of $N=50$, 125, 300, 500, 5000, versus normalized number of transmitted frames K/N.

The number of packets that reach the destination during the time interval $[0,\tau]$ has a Poisson distribution with parameter $-L(\tau,p^*)p^*$. To see this, observe that the number of encoded packets that arrive at the destination form a Poisson process with time-varying intensity $\lambda(s)=\lambda p^* X(s)$, so that

$$\mathbb{E}[N(t,t+dt)] = \lambda \int_0^\tau X(s) p^* ds = -L(\tau,p^*)$$

where p^* is given in (11) and where $L(\tau,p^*)$ is given in (10).

The probability that less than M packets reach the destination is given by

$$P_M(\tau) = \sum_{i=0}^{M-1} \frac{(-L(\tau,p^*)p^*)^i}{i!} \exp(L(\tau,p^*)p^*). \qquad (13)$$

We conclude that $P_s^*(\tau) \geq 1-\delta-P_M(\tau)$.

Finally, we can leverage Corollary 4.2 and obtain the following.

Corollary 5.1: Given p^* as in (11)

$$1-\delta-P_M(\tau) \leq P_s^*(\tau) \leq 1-P_M(\tau).$$

We notice that the bound on the right end of the inequality is obtained by using redundancy as in Section IV for $H \geq M$ then taking the limit of $H \longrightarrow \infty$.

A. Numerical Examples: A Phase Transition

In Fig. 2, we reported on the representation of the bounds described above. The two bounds provide a tight characterization of the performances of fountain codes. In particular, we notice that $P_M(\tau)$, which in fact is the cumulative distribution function (cdf) of a Poisson r.v., tends to 1 as K increases. This causes the success probability to tend to zero as K increases. The intuition is that, for a large number of transmitted frames, the probability of receiving all of them successfully within the given deadline decreases faster than the gain obtained by adding redundancy through fountain codes.

However, the numerical insight of the model says more of the performance attained by fountain codes. In fact, in Fig. 2, we reported P_s^* versus the normalized number of transmitted frames K/N, for increasing N. Interestingly, the success probability becomes more and more close to a step function at the increase of N. We thus observe a phase transition: Above a given number of transmitted frames, the success probability vanishes, and below the same threshold, success occurs with probability 1.

We shall study this phenomenon analytically in Section V-B.

B. Analysis of the Asymptotic Behavior

We now want to understand the behavior of the fountain codes for large values of N in the asymptotic regime $K=K(N) \leq N$. In what follows, we will assume that $\hat{K} := \lim_{N\longrightarrow\infty} K(N)/N$ exists. This implies, of course, that the energy constraint should grow linearly with N, and we thus assume that the limit $\hat{x} := \lim_{N\longrightarrow\infty} x(N)/N$ exists. We can rewrite $-L(\tau,p^*)p^* = N \cdot \Gamma_0^{(0)}$, where we obtain

$$\begin{aligned}\Gamma_0^{(N)} &= -\frac{1}{p^*}\left(1-\lambda p^*\tau - e^{-\lambda p^* \tau}\right)\\ &= \lambda\tau\left(1+\frac{\frac{x}{N-z}}{\log(1-\frac{x}{N-z})}\right)\\ \Gamma_0 := \lim_{N\to\infty}\Gamma_0^{(N)} &= \lambda\tau\left(1+\frac{\hat{x}}{\log(1-\hat{x})}\right).\end{aligned}$$

We have

$$P_M^{(N)}(\tau) = \sum_{i=0}^{K(N)(1+\alpha)-1} \frac{(N\cdot\Gamma_0^{(N)})^i}{i!}\exp(-N\cdot\Gamma_0^{(N)}).$$

We notice that $P_M^{(N)}(\tau) = Pr\{X_N \leq K(N)(1+\alpha)-1\}$, where X_N is a Poisson r.v. From the Strong Law of Large Numbers, we have $\lim_{N\to\infty} X_N/N = \Gamma_0$ P-a.s. Thus

$$\begin{aligned}\lim_{N\to\infty} P_M^{(N)}(\tau) &= 1 - \lim_{N\to\infty} Pr(X_N \geq K(N))\\ &= \begin{cases}1, & \text{if } \hat{K} > \Gamma_0\\ 0, & \text{if } \hat{K} < \Gamma_0\end{cases}\end{aligned}$$

from which we can deduce

$$\lim_{N\longrightarrow\infty} P_s(\tau) = \begin{cases}\geq 1-\delta, & \text{if } \hat{K} < \Gamma_0\\ = 0, & \text{if } \hat{K} > \Gamma_0\end{cases}$$

where we used Corollary 5.1.

This demonstrates the effect of phase transition observed numerically in Fig. 1(c).

VI. Threshold Policies

The way the energy constraints are handled so far is by using only a fraction of the transmission opportunities. This was done uniformly in time by transmitting at a probability that is time-independent; we will refer to this strategy as *static* policy. In this section, we consider an alternative way to distribute the transmissions: We use every possible transmission opportunity till some time limit, and then stop transmitting. This is motivated by the "spray and wait" policy [21], [26] that is known to trade off very efficiently message delay and number of replicas in case of a single message. In general, the optimality of threshold policies has been proven in [5] for general single-frame forwarding policies under the same assumptions adopted here.

In addition, we need to specify the values of the p_i: the probability that packet i is forwarded to a relay when there is an opportunity to transmit (and the time limit has not yet elapsed). We have $\sum_{i=1}^{K} p_i = 1$.

In the lack of redundancy, we recall that the success probability writes

$$P_s(\tau, \mathbf{p}) = \prod_{i=1}^{K} \left(1 - \exp\left(- \lambda \int_0^{\tau} X_i(v) dv \right)\right).$$

Clearly, in our context, transmitting always when there is an opportunity to transmit is optimal if by doing so the energy constraint is not violated. This can be considered to be a trivial time limit policy with a limit of $r = \tau$. Otherwise, the optimal value r for a threshold is the one that achieves the energy constraints, or equivalently, the one for which the expected number of transmissions during time interval $[0, r]$ by the source is x. As before, x is related to the total constraint on the energy through the constraint $\mathcal{E}(\tau) \le \varepsilon x$. The value r of the time limit is then given by $\sigma(z)$; see (9).

Since we considered a time limit policy, $X_i(v)$ first grows till the time limit r is reached, and then it stays unchanged during the interval $(r, \tau]$. For the nontrivial case where $r = \sigma(z)$, we thus have

$$P_s(\tau, \mathbf{p}) = \prod_{i=1}^{K} \Bigg[1 - \exp\Bigg(- \lambda \int_0^{\sigma(z)} N p_i \left(1 - e^{-\lambda v}\right) dv - \lambda(\tau - \sigma(z)) X_i(\sigma(z))\Bigg)\Bigg] = \prod_{i=1}^{K} \tilde{Z}(p_i)$$

where $\tilde{Z}(p_i) := 1 - \exp\left(p_i \tilde{L}(\tau)\right), p_i \tilde{L}(\tau) := -\lambda \int_0^{\tau} X_i(v) dv$, and where we have by (6) and (9)

$$X_i(\sigma(z)) = N p_i \left(1 - e^{-\lambda \sigma(z)}\right) = \frac{N p_i x}{N - z}.$$

Also, the integral can be expressed as

$$\int_0^{\sigma(z)} X_i(v) dv = -\frac{N p_i}{\lambda} \left[-\frac{x}{N - z} + \log\left(1 - \frac{x}{N - z}\right)\right].$$

In particular, the following holds

$$\begin{aligned}\tilde{L}(\tau) &= \frac{Nx}{N - z} + N \log\left(1 - \frac{x}{N - z}\right) - \lambda(\tau - \sigma(z)) \frac{Nx}{N - z} \\ &= -\frac{Nx}{N - z} \lambda \tau - \beta(z)\end{aligned}$$

where

$$\beta(z) = -\frac{Nx}{N - z} - N\left(1 - \frac{x}{N - z}\right) \log\left(1 - \frac{x}{N - z}\right) \ge 0.$$

Finally, the p_i's are selected to be all equal (and to sum to 1) due to the same arguments as in the proof of Theorem 3.1, as $\log(\tilde{Z})$ is concave in its argument here as well.

In the case of redundancy, the calculations are similar to those in (14), where

$$P_s(\tau, H, K) = \sum_{s=K}^{H+K} \binom{H + K}{s} \tilde{Z}^s(p_i)(1 - \tilde{Z}(p_i))^{N-s}. \quad (14)$$

Hence, given $H \ge 0$, it holds

$$\begin{aligned}P_s^*(\tau, H, K) = \exp\left(- \frac{\frac{Nx}{N-z} \lambda \tau + \beta(z)}{H + K}\right) \sum_{s=K}^{H+K} \binom{H + K}{s} \\ \times \left[\exp\left(\frac{\frac{Nx}{N-z} \lambda \tau + \beta(z)}{H + K}\right) - 1\right]^s. \quad (15)\end{aligned}$$

Here again, we used the fact that the success probability is maximized for equal p_i's, i.e.,

$$p_i^*(t) = \begin{cases} 1/(H + K), & \text{if } t \le \sigma(z) \\ 0, & \text{if } t > \sigma(z) \end{cases}$$

and $P_s^*(\tau, H, K)$ is given as in (15). The proof follows the same lines as that of Lemma 4.1.

As a final remark, consider the trivial case where the energy constraint is not active: $\tau < \sigma(z)$, i.e., it is optimal to transmit all the x packets up to time τ. In this case, results from Theorem 4.1 hold.

Now we want to specialize the use of threshold policies in the case of fountain codes when the energy constraint is active: $\tau \ge \sigma(z)$. In this case, the number of packets that reach the destination during the time interval $[0, \tau]$ has a Poisson distribution with parameter $\Lambda = -\tilde{L}(\tau) = Nx/N - z \lambda \tau + \beta(z)$. The probability that less than $M = K \log(K)$ packets reach the destination is given by

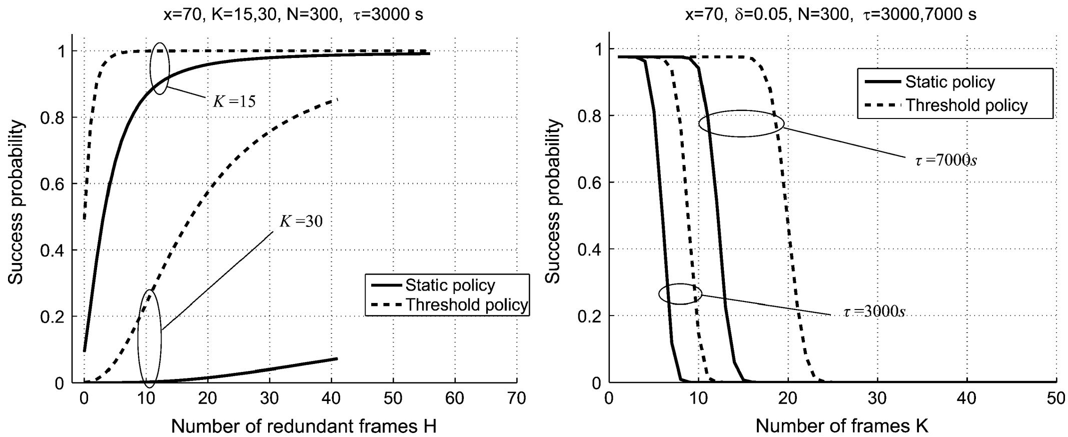
$$\begin{aligned}P_M(\tau) = \exp\left(-\frac{Nx}{N - z} \lambda \tau - \beta(z)\right) \\ \times \sum_{i=0}^{M-1} \frac{1}{i!} \left(\frac{Nx}{N - z} \lambda \tau + \beta(z)\right)^i\end{aligned}$$

and the statement of Corollary 5.1 holds accordingly (notice that when the energy constraint is not active, we fall back to the original form of Corollary 5.1).

A. *Comparison to Static Policies*

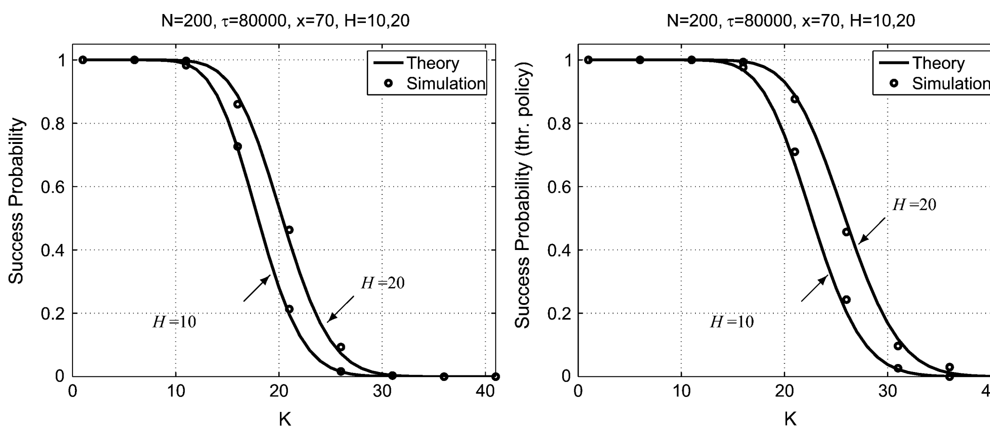
Here, we would like to see what is the relative performance of static and threshold policies both for fountain codes and erasure codes. In Fig. 3(a) and (b), we reported on the case a bound on energy exists for $x = 70$. In such a case, $\sigma(z) = 1000$ s. Concerning erasure codes, when the number of frames is high (30), the usage of a large number of redundant frames proves much more effective compared to static policies. Conversely, for a lower number of frames, the advantage of threshold policies is less marked.

In the case of fountain codes, threshold policies are again more efficient than static policies, and the effect is more relevant for large values of the time constraint τ. We recall that we refer to *optimal* static policies.



(a) (b)

Fig. 3. Comparison of success probability with erasure codes when using threshold and static policies. $N = 300$, $x = 70$, $\tau = 3000, 7000$. (a) Erasure codes, $K = 15, 30$. (b) Fountain codes, $\delta = 0.05$.



(a) (b)

Fig. 4. (a) Simulation results for the success probability of erasure coded messages, static policy, $H = 10, 20$, $N = 200$, $\tau = 80000$ s, $x = 70$. (b) Same as (a), but under threshold policies.

VII. Numerical Validation

In this section, we provide a numerical validation of the model. Our experiments are trace-based. Message delivery is simulated by a MATLAB script receiving prerecorded contact traces as input.

1) Synthetic Mobility: We considered first a Random Waypoint (RWP) mobility model [8]. We registered a contact trace using Omnet++ with $N = 200$ nodes moving on a squared playground of side 5 km. The communication range is $R = 15$ m, the mobile speed is $v = 5$ m/s, and the system starts in steady-state conditions in order to avoid transient effects [17]. The time limit is set to $\tau = 80\,000$ s, which corresponds roughly to 1-day operations, and the constraint on the maximum number of copies is $x = 70$. With this first set of measurements, we want to check the fit of the model for the erasure codes and fountain codes. In the case of erasure codes, we fixed $H = 10$ and 20 and increased the number of message frames K. We selected at random pairs of source and destination nodes and registered the sample probability that the message is received at the destination by time τ. As seen in Fig. 4(a), the fit with the model is rather tight, and an abrupt transition from high success probability to zero is visible. Also, in Fig. 4(b) we reported on the results obtained in case of threshold policies; the fit is similar to what is obtained for static policies, confirming the gain of performance with respect to static policies.

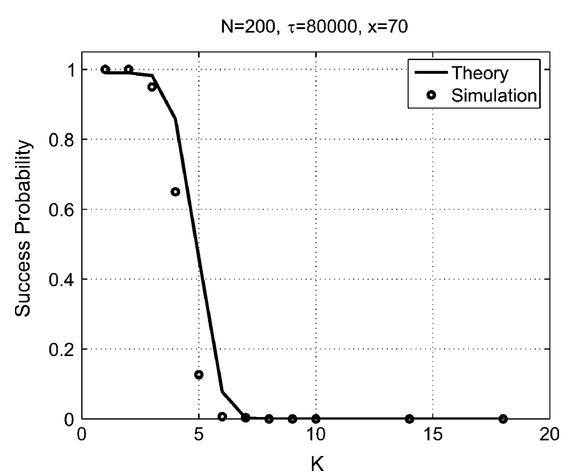


Fig. 5. Simulation results for the success probability of fountain coded messages, static policy, $N = 200$, $\tau = 80\,000$ s, $x = 70$, $\delta = 0.02$.

We repeated the same experiment in the case of fountain codes, as reported in Fig. 5 for static policies. In this case, the

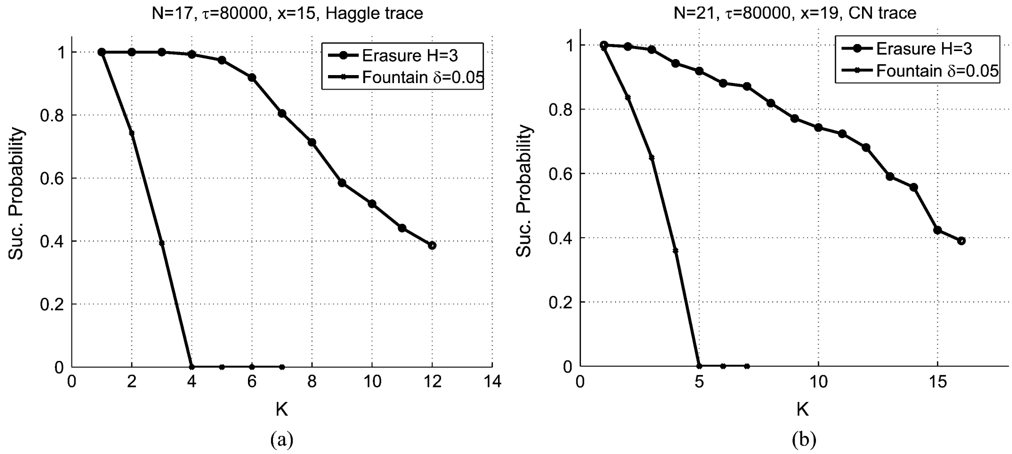


Fig. 6. (a) Simulation results for the success probability of static policies, Haggle trace, $N = 17$, $\tau = 80\,000$ s, $x = 17$, and $\delta = 0.05$. (b) Simulation results for the success probability of static policy, CN trace, $N = 21$, $\tau = 80\,000$ s, $x = 17$, and $\delta = 0.05$.

code-specific parameter is $\delta = 0.02$, and again we increased K. Even in this case, we see that the threshold effect predicted by the model is apparent.

2) Real-World Traces: Our model captures the behavior of a sparse mobile ad hoc network under some assumptions; the most stringent is the uniformity and the stationarity of intermeeting intensities. A tight match with real-world traces is thus out of the scope of our model. However, we would like to draw some qualitative conclusion on the impact of nonuniform and nonstationary encounter patterns seen in those traces.

Haggle: In [9] and related works, the authors report extensive experimentation conducted in order to trace the meeting pattern of mobile users. A version of iMotes, equipped with a Bluetooth radio interface, was distributed to a number of people, each device collecting the time epoch of meetings with other Bluetooth devices. In the case of Haggle traces, due to the presence of several spurious contacts with erratic Bluetooth devices, we restricted the contacts to a subset having experienced at least 50 contacts, resulting in 19 active nodes.

CN: The CN dataset has been obtained by monitoring 21 employees within Create-Net and working on different floors of the same building during a four-week period. Employees volunteered to carry a mobile running a Java application relying on Bluetooth connectivity. The application periodically triggers (every 60 s) a Bluetooth node discovery. Detected nodes are recorded via their Bluetooth address, together with the current timestamp on the device storage for a later processing.

Fig. 6(a) and (b) depicts the results of experiments performed with these data sets. A major impact is played by the nonstationarity of traces. This is mainly due to "holes" appearing in the trace that impose unavoidable cutoff effects on the success probability. In particular, in the case of fountain coding, for $K > 1$, i.e., when the original file is actually fragmented, the performance is rather poor. This is due to the increase with K of the number of frames M required for decoding, exacerbated by the reduced size of the network. For example, for $K = 3$ and $\delta = 0.05$, it holds $M = 13$, i.e., M is close to N for the Haggle trace and larger than $N/2$ for the CN trace. This means that, in order to deliver the message, basically almost all nodes should meet the destination and deliver a frame within τ.

With erasure codes, both traces show the characteristic cutoff of performance at the increase of K. The decay, though, depends on the trace considered. In particular, a close look to the two data sets showed that the Haggle trace has higher average intermeeting intensity compared to the CN trace over the considered interval. The shape seen in Fig. 6(a) more closely resembles the theoretical sigmoid cutoff predicted by the model.

We can then draw two conclusions. First, in the case of real-world traces, we can expect again a threshold-type decay of the success probability for a large number of nodes. However, the transition that is measured for a small number of nodes is not abrupt in the case of erasure codes. The use of fountain codes to obtain high delivery probability, conversely, may be limited in practical cases to a small number of frames, i.e., when the files to be split are not too large.

VIII. Conclusion and Remarks

We have considered in this paper the tradeoff between energy and probability of successful delivery in presence of finite duration of contacts or limited storage capacity at a node. It can store only part (a frame) of a file that is to be transferred. To improve performance, we considered the generation of erasure codes at the source that allows the DTN to gain in spatial storage diversity. Both fixed erasure codes (Reed–Solomon-type codes) as well as rateless fountain codes have been studied.

Fountain codes can be viewed as a special case of general network coding such as those studied in [10], [12], and [18]. We note, however, that in order to go beyond fountain codes (in which coding is done at the source) and consider general network coding (where coding is also done in the relay nodes), one needs not only change the coding approach, but one should allow storage of several frames at each relay node.

Finally, an important issue addressed in this work is the existence of phase transitions in the coding schemes used to overcome disconnections in DTNs. It indicates that, when scaling up the number of nodes in the network, one is faced with abrupt changes in performance depending on the ratio of the number of redundant frames introduced and the number of information frames to be received. Furthermore, this tradeoff, represented by threshold-type formulas for the success probability, is directly related to a minimal set of parameters: the product $\lambda\tau$ and the energy spent, i.e., the number of messages x (under a linear energy model). We also showed how to tie such parameters to the complexity of coding operations. This result reveals fundamental constraints for any similar scheme to be applied in practice.

Appendix

A. Proof of Theorem 3.1

Proof: The case $p = 0$ is trivial, so we assume below $p > 0$. We have

$$P_s^*(\tau) = \max_{\mathbf{p}:\sum_i p_i = p} P_s(\tau) = \max_{\mathbf{p}:\sum_i p_i = p} \prod_{i=1}^{K} Z(p_i).$$

$\mathbf{p}^*$ is optimal if and only if it maximizes $g(\mathbf{p}) := \sum_{i=1}^{K} \log(Z(p_i))$ s.t. $\sum_i p_i = p$, $p_i \geq 0$. Note that $L(\tau)$ is nonpositive. $\log Z$ turns out to be a concave function. It then follows from Jensen's inequality that

$$\begin{aligned} g(\mathbf{p}) &= \frac{1}{K}\sum_{i=1}^{K} \log(Z(p_i)) \\ &\leq \log\left(Z\left(\frac{1}{K}\sum_{i=1}^{K} p_i\right)\right) = \log\left(Z\left(\frac{p}{K}\right)\right) \end{aligned} \quad (16)$$

where equality is attained for $p_i = p/K$. This concludes the proof. ■

B. Proof of Proposition 4.2

Proof: From Theorem 4.1

$$P_s^*(\tau, K, H) = P\{S_{H+K,\hat{p}} \leq H + K\} - P\{S_{H+K,\hat{p}} < K\}$$

where $\hat{p} = 1 - \exp\left(\frac{L(p^*,\tau)p^*}{H+K}\right)$.

We can rewrite $-L(\tau, p^*)p^* = N \cdot \Gamma_0^{(0)}$, where we obtain

$$\begin{aligned} \Gamma_0^{(N)} &= -\frac{1}{p^*}\left(1 - \lambda p^*\tau - e^{-\lambda p^*\tau}\right) \\ &= \lambda\tau\left(1 + \frac{\frac{x}{N-z}}{\log\left(1 - \frac{x}{N-z}\right)}\right) \end{aligned}$$

and we define

$$\Gamma_0 := \lim_{N\to\infty} \Gamma_0^{(N)} = \lambda\tau\left(1 + \frac{\hat{x}}{\log(1-\hat{x})}\right).$$

Notice also that

$$\begin{aligned} E[S_{H+K,\hat{p}}] &= (H+K)\hat{p} \\ &= (H+K)\left(1 - e^{-\frac{N\Gamma_0^{(N)}}{H+K}}\right) \\ &= (H+K)\left(\frac{N\Gamma_0^{(N)}}{H+K} + o\left(\frac{1}{H+K}\right)\right) \end{aligned}$$

from which it follows that

$$\lim_{N\longrightarrow\infty} \frac{E[S_{H+K,\hat{p}}]}{N} = \lim_{N\to\infty} (H+K)\hat{p}/N = \Gamma_0.$$

Notice that the binomial r.v. $S_{H+K,\hat{p}}$ can be interpreted as the sum of $H+K$ independent binary random variables i_n with mean $\hat{p}$ and variance $\hat{p}(1-\hat{p})$. Also, the series of the normalized variances $\sigma_{i_n}^2 = \frac{1}{n^2}\hat{p}(1-\hat{p}) \leq \frac{1}{n^2}$ is finite. Thus, the Strong Law of Large Numbers [11] ensures that $\lim_{N\to\infty} S_{H+K,\hat{p}}/N = \Gamma_0$ P-a.s.

Now, we can derive

$$\lim_{N\to\infty} P\{S_{H+K,\hat{p}} \leq H+K\} = \begin{cases} 0, & \text{if } \hat{K} + \hat{H} \leq \Gamma_0 \\ 1, & \text{if } \hat{K} > \Gamma_0 \end{cases}$$

and, in a similar way

$$\lim_{N\to\infty} P\{S_{H+K,\hat{p}} < K\} = \begin{cases} 0, & \text{if } \hat{K} < \Gamma_0 \\ 1, & \text{if } \hat{K} \geq \Gamma_0 \end{cases}.$$

Finally, by enumeration

$$\lim_{N\to\infty} P_s(\tau, K, H) = \begin{cases} 0, & \text{if } \hat{K} + \hat{H} \leq \Gamma_0 \\ 0, & \text{if } \hat{K} + \hat{H} > \Gamma_0 \text{ and } \hat{K} \geq \Gamma_0 \\ 1, & \text{if } \hat{K} + \hat{H} > \Gamma_0 \text{ and } \hat{K} < \Gamma_0. \end{cases}$$

This concludes the proof. ■

References

[1] J. J. Metzner, "An improved broadcast retransmission protocol," *IEEE Trans. Commun.*, vol. COMM-32, no. 6, pp. 679–683, Jun. 1984.

[2] C. Huitema, "The case for packet level FEC," in *Proc. IFIP*, Sophia Antipolis, France, Oct. 1996, pp. 109–120.

[3] J. Kurose, D. Towsley, and D. Rubenstein, "Real-time reliable multicast using proactive forward error correction," in *Proc. IEEE NOSSDAV*, Cambridge, U.K., Jul. 8–10, 1998.

[4] J. Nonnenmacher, E. Biersack, and D. Towsley, "Parity-based loss recovery for reliable multicast transmission," *IEEE/ACM Trans. Netw.*, vol. 6, no. 4, pp. 349–361, Aug. 1998.

[5] E. Altman, T. Basar, and F. De Pellegrini, "Optimal monotone forwarding policies in delay tolerant mobile ad-hoc networks," *Perform. Eval.*, vol. 67, no. 4, pp. 299–317, 2010.

[6] B. Bollobás*, Random Graphs*. Cambridge, U.K.: Cambridge Univ. Press, 2001.

[7] S. Burleigh, L. Torgerson, K. Fall, V. Cerf, B. Durst, K. Scott, and H. Weiss, "Delay-tolerant networking: An approach to interplanetary Internet," *IEEE Commun. Mag.*, vol. 41, no. 6, pp. 128–136, Jun. 2003.

[8] T. Camp, J. Boleng, and V. Davies, "A survey of mobility models for ad hoc network research," *Wireless Commun. Mobile Comput.*, vol. 2, no. 5, pp. 483–502, Aug. 2002.

[9] A. Chaintreau, P. Hui, J. Crowcroft, C. Diot, R. Gass, and J. Scott, "Impact of human mobility on opportunistic forwarding algorithms," *IEEE Trans. Mobile Comput.*, vol. 6, no. 6, pp. 606–620, Jun. 2007.

[10] A. E. Fawal, K. Salamatian, D. C. Y. Sasson, and J. L. Boudec, "A framework for network coding in challenged wireless network," in *Proc. MobiSys*, Uppsala, Sweden, Jun. 19–22, 2006.

[11] W. Feller, *Introduction to Probability Theory and Its Applications*, 3rd ed. New York: Wiley, 1971.

[12] C. Fragouli, J.-Y. L. Boudec, and J. Widmer, "Network coding: An instant primer," *Comput. Commun. Rev.*, vol. 36, no. 1, pp. 63–68, 2006.

[13] R. Groenevelt and P. Nain, "Message delay in MANET," in *Proc. ACM SIGMETRICS*, Banff, AB, Canada, Jun. 6, 2005, pp. 412–413.

[14] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, no. 4, pp. 477–486, Aug. 2002.

[15] S. Jain, M. Demmer, R. Patra, and K. Fall, "Using redundancy to cope with failures in a delay tolerant network," *Comput. Commun. Rev.*, vol. 35, no. 4, pp. 109–120, 2005.

[16] T. Karagiannis, J.-Y. L. Boudec, and M. Vojnovic, "Power law and exponential decay of inter contact times between mobile devices," in *Proc. ACM MobiCom*, Montréal, QC, Canada, Sep. 9–14, 2007, pp. 183–194.

[17] J.-Y. L. Boudec and M. Vojnovic, "Perfect simulation and stationarity of a class of mobility models," in *Proc. IEEE INFOCOM*, Miami, FL, Mar. 13–17, 2005, pp. 183–194.

[18] Y. Lin, B. Liang, and B. Li, "Performance modeling of network coding in epidemic routing," in *Proc. MobiSys MobiOpp*, San Juan, Puerto Rico, Jun. 11, 2007, pp. 67–74.

[19] D. J. MacKay, *Information Theory, Inference, and Learning Algorithms*. Cambridge, U.K.: Cambridge Univ. Press, 2003.

[20] T. Spyropoulos, K. Psounis, and C. Raghavendra, "Efficient routing in intermittently connected mobile networks: The multi-copy case," *IEEE/ACM Trans. Netw.*, vol. 16, no. 1, pp. 77–90, Feb. 2008.

[21] T. Spyropoulos, K. Psounis, and C. S. Raghavendra, "Spray and wait: An efficient routing scheme for intermittently connected mobile networks," in *Proc. ACM SIGCOMM WDTN*, Philadelphia, PA, Aug. 26, 2005, pp. 229–236.

[22] Y. Wang, S. Jain, M. Martonosi, and K. Fall, "Erasure-coding based routing for opportunistic networks," in *Proc. ACM SIGCOMM WDTN*, Philadelphia, PA, Aug. 26, 2005, pp. 229–236.

[23] J. Widmer and J.-Y. L. Boudec, "Network coding for efficient communication in extreme networks," in *Proc. ACM SIGCOMM WDTN*, Philadelphia, PA, Aug. 26, 2005, pp. 284–291.

[24] X. Zhang, G. Neglia, J. Kurose, and D. Towsley, "Performance modeling of epidemic routing," *Comput. Netw.*, vol. 51, pp. 2867–2891, Jul. 2007.

[25] A. Weiss and A. Shwartz, *Large Deviations for Performance Analysis, ISBN 0-412-06311-5*. London, U.K.: Chapman & Hall, 1995.

[26] T. Spyropoulos, T. Turletti, and O. Obraczka, "Routing in delay tolerant networks comprising heterogeneous node populations," *IEEE Trans. Mobile Comput.*, vol. 8, no. 8, pp. 1132–1147, Aug. 2009.

[27] C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*. Cambridge, U.K.: Cambridge Univ. Press, 2003.

[28] M. G. Luby, M. Mitzenmacher, M. A. Shokrollahi, and D. A Spielman, "Efficient erasure correcting codes. Information Theory," *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 569–584, Feb. 2001.

Eitan Altman (SM'00–F'10) received the B.Sc. degree in electrical engineering, the B.A. degree in physics, and the Ph.D. degree in electrical engineering from the Technion—Israel Institute of Technology, Haifa, Israel, in 1984, 1984, and 1990, respectively, and the B.Mus. degree in music composition from Tel-Aviv University, Tel-Aviv, Israel, in 1990.

Since 1990, he has been a Researcher with the National Research Institute in Informatics and Control (INRIA), Sophia-Antipolis, France. He is on the Editorial Boards of several scientific journals: *Wireless Networks*, *Computer Networks*, *Journal of Discrete Event Dynamic Systems*, *SIAM Journal of Control and Optimization*, *Stochastic Models*, and *Journal of Economy Dynamic and Control*.

Dr. Altman has been the (co)chairman of the program committee of international conferences, including the Workshop on Modeling and Optimization in Mobile, Ad Hoc and Wireless Network (WiOpt 2003); International Symposium on Dynamic Games and Applications (ISDG 1998); and Valuetools (2006). He was General Chair of WiOpt 2005. He has been a plenary speaker in various international conferences. He received the Best Paper Award in the Networking 2006 conference and is a coauthor of two papers that have received the Best Student Paper awards at QoFis 2000 and Networking 2002.

Francesco De Pellegrini (M'09) received the Laurea and Ph.D. degrees in telecommunication engineering from the University of Padova, Padova, Italy, in 2000 and 2004, respectively.

He spent 2001–2002 at Boston University, Boston, MA, as a Visiting Scholar. He is currently a Senior Researcher with Create-Net, Milan, Italy. His research interests are location detection, multirate systems, routing, wireless mesh networks, VoIP, and ad hoc and delay-tolerant networks.

Dr. De Pellegrini is a Technical Program Committee member of the IEEE INFOCOM and a reviewer for several international networking conferences and journals. He serves in the Steering Program Committee of the ICST MobiQuitous. He was the Vice-General Chair for the first ICST Robocomm.