Computer Communications 34 (2011) 1645–1659

Contents lists available at ScienceDirect

Computer Communications

journal homepage: www.elsevier.com/locate/comcom

Framework for optimizing the capacity of wireless mesh networks

Christelle Caillouet [a,*], Stéphane Pérennes [b], Hervé Rivano [c]

[a] DRAKKAR Team, LIG Lab, Grenoble, F-38402, France
[b] MASCOTTE Team, I3S Lab (CNRS/UNS)-INRIA, Sophia Antipolis F-06902, France
[c] SWING Team, CNRS, Université de Lyon, INRIA, INSA-Lyon CITI F-69621, France

ARTICLE INFO

Article history:
Received 31 March 2010
Received in revised form 25 February 2011
Accepted 2 March 2011
Available online 12 March 2011

Keywords:
Wireless mesh networks
Capacity
Routing
Scheduling
Linear programming

ABSTRACT

In this paper, we address the problem of computing the transport capacity of Wireless Mesh Networks (WMNs) dedicated to Internet access. Routing and transmission scheduling have a major impact on the capacity provided to the clients. A cross-layer optimization of these problems allows the routing to take into account contentions due to radio interference.

We present a generic mixed integer linear programing description of the configurations of a given WMN, addressing gateway placement, routing, and scheduling optimizations. We then develop new optimization models that can take into account a large variety of radio interference models, and QoS requirements on the routing. We also provide efficient resolution methods that deal with realistic size instances. It allows to work around the combinatoric of simultaneously achievable transmissions and point out a critical region in the network bounding the network achievable capacity. Based upon strong duality arguments, it is then possible to restrict the computation to a bounded area. It allows for computing solutions very efficiently on large networks.

© 2011 Elsevier B.V. All rights reserved.

1. Introduction

Wireless mesh networks (WMNs) are cost-effective solutions for ubiquitous high-speed services [1]. They are dynamically self-organized and self-configured networks in which nodes automatically establish and maintain mesh connectivity among themselves. Each node, called mesh router, operates not only as a host but also as a router for the traffic. Mesh routers are interconnected in order to form a fixed infrastructure offering connectivity to mesh clients and gateway functionality for connections to the Internet of the mobile network users. This infrastructure, forming a wireless backhaul network, is integrated with the Internet by special routers called mesh gateways. Mesh clients access the Internet by multi-hop communications through the backhaul network.

A WMN can thus be represented as a two-tier architecture as depicted in Fig. 1, and a logical separation is maintained between client-to-router one-hop connections and router-to-gateway multi-hop communications. In this article, we focus on the WMN backhaul, more precisely on computing optimal stationary behaviors. Therefore, the traffic of the mobile clients is modeled by a demand distribution over the mesh routers.

These multi-hop networks are expected to carry high throughput. A very important requirement for the network performance evaluation is to maximize its capacity, i.e. the throughput offered to each flow. This performance measure is however affected by many factors such as network topology, traffic pattern, resource sharing and radio interferences. One of the major issue of wireless networks is the capacity reduction due to radio interferences [2,3].

In order to maximize the capacity of a WMN, one has to determine an optimal allocation of the shared resources, in particular the radio channel available. This allocation is constrained by interferences and contentions around a communicating node. If many concurrent transmissions are successful, they have to be pairwise non interfering. Contending links share a common channel using a time multiplexing process. In this TDMA scheme (time-division-multiple-access), the transmission capacity is divided into time slots, and each link is assigned some dedicated slots.

The goal of this paper is to provide optimization-based models and frameworks determining bounds on the optimal theoretical capacity of a WMN. Capacity evaluation with a cross-layer approach is useful to improve the performance of routing protocols and MAC layers. Routing efficiency depends on the problem of allocating physical and data link layer resources, since interferences between link transmissions clearly impact the routing used to satisfy traffic demands. Similarly, traffic routing determines the traffic flow requirements on each link, according to the achievable capacity of the links.

Following the state of the art of existing optimization techniques developed for WMN, we present a cross-layer model for optimal wireless mesh network configurations. We develop in Sections 3 and 4 a global mixed-integer linear description of gateway

* Corresponding author. Tel.: +33 0 476 827 219; fax: +33 0 476 827 287.
E-mail address: christelle.molle-caillouet@imag.fr (C. Caillouet).

0140-3664/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.comcom.2011.03.002

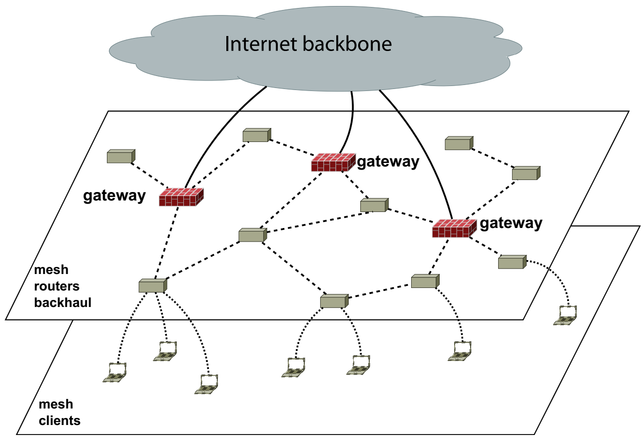


Fig. 1. A WMN topology: Mobile clients access Internet through a multi-hop wireless backhaul network of routers and gateways.

placement, routing, and scheduling optimizations. Motivated by an efficient computation of the optimal theoretical capacity of the network, we focus on the joint optimization of routing and scheduling and, in the case of a steady state operating network, its efficient relaxation known as the Round Weighting Problem (RWP) in Section 5. Our contribution is to develop linear programming formulations of this problem and a column generation process which allows to get throughput optimal, yet fractional, routing and round weighting efficiently.

We focus on router-to-gateway traffic pattern, for which the routing can be formulated as a single-commodity flow problem that we want to maximize. In graph theory, the maximum flow problem is known to be the dual of the minimum cut problem, with equal optimal solutions [4].

A new representation of the RWP allows us to consider the activation duration of links in such a way that traffic can cross the network cuts. In other words, we investigate the RWP in which we eliminate the routing and focus on the transport capacity available on the network cuts. In Section 6, we present our contribution on this problem, that is, a linear formulation involving cuts and rounds in the WMN whose optimal solutions are proved to be equivalent to the previous formulations of RWP. To solve this new formulation efficiently, we have developed a primal–dual algorithm using a cross row and column generation process. We analyze the simulation results of this formulation, highlighting the presence of a bottleneck area in the network that restricts the available capacity.

2. Related work

Many challenges on WMNs are still open, because finding a model that combines specific constraints of wireless mesh technology and topology is not easy. The study of wireless network performances has motivated many research works. WMNs deployment in operational situations such as urban areas requires quality of service (QoS) criteria that are challenging to guarantee. Indeed, recent works have pointed out fundamental issues with capacity and scalability.

In [5], Gupta and Kumar presented a pioneering study of the capacity of wireless ad-hoc networks. Under specific routing, radio interference models and probabilistic traffic assumptions, they use an analytical approach to prove that the per node capacity of a random wireless ad-hoc network decreases as $\mathcal{O}(\frac{1}{\sqrt{n}})$ as the size of the network, n, grows. Other works tried to give more analysis and refine this result [6–8], but all of them confirm that the network capacity goes down when its size increases. Unlike ad-hoc networks, WMNs are stationary networks in which traffic is mainly router-to-gateway (respectively gateway-to-router) oriented. The available capacity for each node is therefore reduced to $\mathcal{O}(\frac{1}{n})$ [9,10].

To determine the optimal resource allocation, MAC protocols achieving conflict-free link scheduling have been developed to avoid interferences [2,11]. In order to deal with interference, it is important to know what are the sets of transmissions that can be active at the same time. An algorithm enumerating a tractably large subset of simultaneous transmissions has been developed in order to compute an approximated solution for maximum throughput using linear programming [12]. For a multi-radio multi-channel network problem, a column generation approach has been developed for minimizing the scheduling time [13]. Optimal and near-optimal solutions have been computed also using column generation for resource allocation in ad hoc networks where each node must be activated in a minimum time period [14]. Another approach consists in solving a multi-commodity flow problem by a dual method in order to find upper bounds for the achievable throughput [3,15]. A survey on efficient algorithms for resource allocation in large wireless networks shows the efficiency of column generation compared to complete enumeration for the scheduling problem [16]. Our approach does not limit the use of column generation for scheduling constraints but includes also the routing for considering QoS.

Several interference models exist in the literature. An investigation of the impact of the choice of the interference model, on the conclusions that can be drawn regarding the performance of wireless networks, by comparing different interference models has been presented in [17]. Our approach is to provide optimization models so that any existing interference model can be plugged in transparently.

The impact of interferences on routing has also been investigated [2,18]. The routing has been deeply investigated in multi-hop wireless networks, but joint routing and scheduling optimization in WMNs is a more recent topic motivated by the efficiency of cross layer approaches [19]. It consists in computing jointly the router-gateway routes of the packets, and the transmissions schedule in order to achieve the maximum transport capacity. This problem is closely related to the optimal assignment of interference-free broadcasting schedules in multi-hop packet radio networks, which has been shown to be NP-hard [20]. Another related problem, in slightly different settings, is the *minimum time gathering* (MTG) [21,22] where each node has to send one unit of data to a central node within a global minimum gathering time. A 4-approximation algorithm has been developed for this problem [23]. Routing and scheduling in WMNs with delay constraints has also been addressed in [24].

Column generation processes dealing with the joint routing and scheduling problem have been presented. A nonlinear approach for general wireless networks is presented in [25]. The approach deals with a different objective and cannot be solved for networks with more than 20 nodes. For wireless mesh networks, column generation has been introduced in order to solve the routing problem where multi-routes, as well as router to multi-gateway traffic, are not allowed [13,26]. In these works, the columns represent sets of pairwise non interfering transmissions. However, several works have shown that multi-path routing increases the end-to-end network throughput [27], and a multi-gateway association results in better capacity and fairness in the network [28]. Our linear programs add the possibility of generating paths with QoS constraints in princing subproblems, e.g. by limiting the length (in terms oh hop) of the eligible paths in the routing. By reducing the search space for paths, while not reducing any combinatorial overcharge, this can even decrease the complexity of the problem (see Section 5.3).

A comparison between CP-based and column generation has been done in [29]. The column generation process to generate sets of pairwise compatible links for the scheduling has also been considered for the channel assignment problem in WMNs [30]. Even if the models presented in these study are similar to the one presented here, the purpose of our paper is to discuss about the gateway placement problem in WMNs according to results given by a new formulation of the joint routing and scheduling problem involving cuts arounds the mesh gateways.

In the literature, some papers have investigated the optimization of the deployment cost and the quality of service (QoS) achieved by carefully placing the gateways in the network [31]. This problem is closely related to the facility location problem which seeks to minimize the length of the paths from any node to the set of "facilities" [32].

The gateway placement problem has also been studied for different kinds of networks. In wireless sensor networks, authors of [33] study how to place multiple gateways in an area such that the data collection latency is minimized. They transform the placement problem into a cluster assignment problem, minimizing the total number of forwarding operations in each cluster. In [34], authors address the problem of optimally placing one or multiple gateways in both 1-D or 2-D vehicular networks to minimize the average number of hops from routers to gateways so the communication delay can be decreased. Cost consideration are addressed in [35] to optimally select gateway in a cellular Wi-Fi network. Integer linear programming and heuristic solution are proposed to guarantee minimum hop distance to the gateway from all access points.

As a matter of fact, in many scenarios the location of such special nodes would have a strong impact on the network performances. This question has fostered several studies in WMN optimization [36–38]. In [36], authors used a grid-based approach to place k gateways in a wireless mesh networks so that the total throughput achieved by interference-free scheduling is maximized. They divide the network into k cells and place a gateway at the center of each cell, obtaining an approximation of the optimal solution. A polynomial time approximation algorithm using a cluster-based approach that finds a near optimal gateway placement is also presented in [37]. A study focusing of the placement of one gateway in a grid network highlights that a better throughput is obtained if the gateway is near the center of the grid [38]. Our experiments in Section 5.4 moderate the impact of the gateway placement in the special case of wireless mesh networks in which the traffic is gathered by the gateways. We claim that there are specific conditions on the network topology that are sufficient to guarantee an efficient gateway placement.

3. Model and assumptions

The fixed infrastructure of the WMN is modeled by a directed graph $G = (V,E)$ of N nodes representing mesh points, i.e. the union of mesh routers and gateways. Without loss of generality, we assume that each device has a single interface allowing to send or receive packets on the only channel available[1]. This channel is therefore shared between all the nodes. The set of vertices V is decomposed into two non-intersecting subsets: $V = V_r \cup V_g$, with $V_r \cap V_g = \emptyset$, where V_g is the set of mesh gateways and V_r the set of mesh routers that do not interact directly with Internet (see Fig. 2).

As opposite to mobile ad-hoc networks (MANETs) in which every node can potentially communicate pairwise together, the traffic flow in WMNs is mainly router-to-gateway oriented (respectively gateway-to-router). We therefore assume that each router r of V_r has a demand d_r to send to the gateways, corresponding to the aggregated traffic of the mesh clients connected to it.

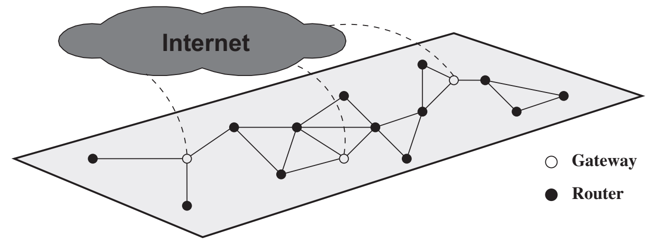


Fig. 2. The connectivity graph $G = (V,E)$ modeling a WMN: White nodes are gateways V_g and black nodes are routers V_r.

The set of links E strongly depends on the physical layer model chosen, that specifies whether or not a packet is received error-free. The existing communication models are usually divided into two classes: Power-based and distance-based models. We overview these models in the next subsection and define at the same time the choice we made for this work.

3.1. Communication settings

The WMN is assumed to be a single channel synchronous network. All the network nodes operate with a single omnidirectional antenna. The quality of a communication between two nodes in the network is affected by many factors and measured by the *signal to noise ratio* (SNR) that depends on the power of the sent signal affected by the power of the ambient noise:

$$SNR_v = 10\log_{10}\frac{P(signal)}{P(noise)} \quad (dB)$$

where $P(signal)$ (resp. $P(noise)$) represents the transmission power, or sent signal power (resp. the noise power). The signal power at a receiving node usually depends on the transmitting power of the sending node, and the path-loss effects depending on the distance between the sender and the receiver. In this model, each node in the network has a probability of well-receiving the message sent, and this probability tends to zero at infinity.

This power-based model is commonly approximated by a threshold: If the SNR is above a given threshold, then the signal is well-received by the receiver, otherwise the communication fails. This model is validate by information theory that says that it is always possible to get a threshold model by properly choosing the channel coding [39].

The physical model used in many research works is a distance-based model, that is based on the location of the nodes in the plane considered. Approximating a path-loss model, one can consider that a transmission between two nodes is possible if they are within communication range of each other. This range depends on the euclidean distance between the two nodes.

We therefore use a threshold-based model (modeling either a model with euclidean distance or a SNR with threshold and uniform power), and we model for each node the set of its immediate neighbors, i.e. the set of nodes it can directly communicate in one hop. We thus obtain the definition of the set of possible transmissions between any pair of nodes modeled by a set of links E in the connectivity graph modeling the WMN.

We assume the wireless communication medium to be *half-duplex*, that is, a node cannot listen to the same channel on which it is transmitting. We impose the constraint of not activating more than one incident link per node:

[1] Considering multi-interface multi-channel WMNs only changes the contention model. As explained in Section 3.2, our interference model is generic so that the extension with several channels is straitforward. In addition, we believe that it does not change the form of the critical behaviour of the network presented in this paper.

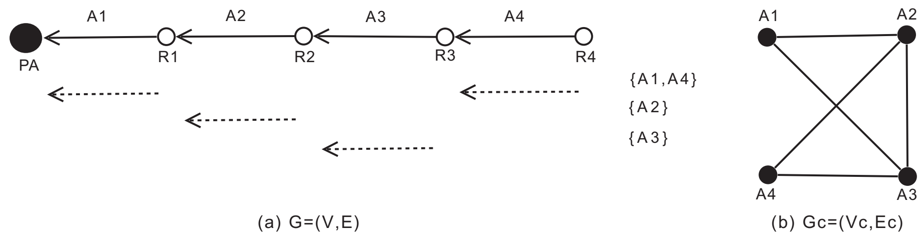


Fig. 3. Distance-2 interference model: (a) Topology graph, (b) Conflict graph. Sets of compatible links $\{A1, A4\}, \{A2\}, \{A3\}$ correspond to independent sets of the conflict graph.

$$\sum_{e \in \Gamma(v)} a(t,e) \leqslant 1, \quad \forall t \leqslant T$$

where $\Gamma(v)$ is the set of links in G incident to node $v \in V$, $a(t,e)$ determines if link e is transmitting at time t or not, and T is the time period length.

Spatial re-use is the opportunity of allowing various simultaneous interference-free transmissions if they are located far enough in the network. To consider spatial re-use, an interference model must be defined. We present existing interference models and define a generic one for our models, allowing to apply simple binary interference models as well as more challenging SINR models.

3.2. Generic interference model

To ensure transmissions during a time slot, links activated have to be pairwise interference-free. The structure of these sets depends on both the wireless technology used by the network devices and the radio interferences. In the literature, existing communication models can be divided into two classes, namely protocol and physical communication models strongly related on the distance-based and power-based models described in the previous section [17,40]. The MAC layer protocol CSMA/CA implemented in 802.11 may use RTS/CTS messages for a pair of nodes to reserve the channel and transmit. Many interference models have been developed on top of CSMA/CA. They are called binary models, meaning that two nodes can either always or never transmit simultaneously unless they communicate together [41]. These binary models can be represented by a conflict graph $G_c = (V_c, E_c)$, where a node in V_c corresponds to a link of E in G, and a link between two nodes of V_c exists if and only if the corresponding links are interfering in G.

A known interference scheme, the symmetric distance-2 interference model, has been inspired by CSMA/CA. We assume that all communications involve bidirectional messages exchanges (RTS-CTS and DATA-ACK exchanges) inducing a symmetric interference pattern. When a router transmits, all its neighbors keep silent. For sake of symmetric transmissions, the same happens with the receiver, inducing incompatibility patterns with the 2-hop neighborhood of an activated link. An example of this model and its associated conflict graph is depicted in Fig. 3.

Another class of interference models are based on the wireless physical layer technologies and the SNR presented before. In WMNs where simultaneous transmissions can occur, one has to integrate the cumulative effects of these transmissions as noise degrading the measured signal. A transmission is successful if the signal-to-interference-plus-noise ratio (SINR) at the receiver remains greater than a fixed threshold throughout the duration of the packet transmission. Formally, the following SINR condition must hold at the receiver:

$$SINR_v = \frac{P_u G_{uv}}{\eta_v + \sum_{w \neq u} P_w G_{wv}} \geqslant \gamma, \qquad (1)$$

where (u, v) is the link of interest, P_v is the transmitting power used by node v, η_v is the thermal noise power at node v, and G_{uv} denotes the channel gain from u to v.

In the literature, another kind of interference model is commonly used. Based on a graph representation of interferences, they are denoted *binary interference models*. A generic writing of these models considers $\mathcal{I}(e)$, the set of links interfering with link e of E in the network. If $a(t,e)$ equals *1* when a link e is activated at time t, the following constraint has to be respected:

$$a(t,e) + a(t,e') \leqslant 1, \quad \forall e' \in \mathcal{I}(e), \quad t \leqslant T \qquad (2)$$

In the remainder of this section, we present our problems using the binary interference model, but the SINR model can be used as shown in section 5.3.

3.3. Traffic requirements

At each time period, a mesh router r sends d_r units of traffic in the network to Internet through the gateways. But the traffic is not enforced to reach the gateways in one period of time. Indeed, routing even one unit of traffic up to the gateways can need several periods. Given a link scheduling during one period, the router's demand can be stored at intermediate nodes. Moreover, all routers have infinite memory, meaning that we do not consider saturating networks where nodes are bottlenecks.

To ensure that data are sent from the routers to the gateways, all links of a path between the sources and the destinations have to be activated during the time period. An active path is then a path whose links can be activated during the time period considered. The flow rate going through an active path is limited by its most constrained link, i.e. the link with lowest activation during the time period. Then, when λ units of flow are sent on an active path during period $[1, T]$, the flow rate of the active path is equal to λ/T in steady state, i.e. when repeating the period among time. Routing at maximum rate then determines a set of paths between the sources and destinations whose links can be scheduled in a minimum period of time.

We focus on router-to-gateway traffic pattern in order to route the demand from the network clients to Internet through the gateways. We first assume that the placement of the gateways V_g is not known *a priori* and has to be determined concurrently with routing and link scheduling. A common traffic model is to consider multicommodity flow constraints, as presented in the next subsection.

3.4. Problem formulation

In Table 1, we recall the notations used in our linear programming formulation addressing gateway placement, routing and scheduling in WMNs. The linear constraints that define the feasibility region of our problems are:

$$a(t,e) + a(t,e') \leqslant 1, \quad \forall e' \in \mathcal{I}(e), \quad t \leqslant T \qquad (3)$$

$$\sum_{v \in V} f(v,e) \leqslant \sum_{t \leqslant T} a(t,e), \quad \forall e \in E \qquad (4)$$

Table 1
Synthesis of the variables and constants used.

Notation	Definition
$d_v \in \mathbb{R}^+$	Traffic demand of mesh router v.
$a(t,e) \in \{0,1\}$	Activation of link e during time slot t.
$s(i) \in \{0,1\}$	Selection of node i to become a gateway.
$f(v,e) \in \mathbb{R}^+$	Flow sent by router v through link e.
$y(r,v) \in \mathbb{R}^+$	Flow sent by router r received by gateway $v \cdot y(r,v) = 0$ if v is not a gateway.
$C \in \mathbb{R}^+$	Constant greater than the total demand in the WMN.
$\mathbb{1}_{\{v=r\}} \in \{0,1\}$	Indicator function: 1 if $v = r$, 0 otherwise.

$$\sum_{e=(u,v)\in E} f(r,e) + \mathbb{1}_{\{v=r\}} d_v = \sum_{e=(v,w)\in E} f(r,e) + y(r,v), \quad \forall r, \quad v \in V \tag{5}$$

$$\sum_{i\in V} y(r,i) = d_r, \quad \forall r \in V \tag{6}$$

$$y(r,i) \leqslant s(i) \cdot C, \quad \forall i \in V, \quad r \in V \tag{7}$$

Multi-commodity flow aims at computing paths between mesh routers and gateways in the single channel WMN considered. Multi-commodity flow constraints express the injection of traffic d_r by router r into the network that is forwarded without loss by other mesh routers until its extraction by the gateways. Recall that $f(r,e)$ is the continuous flow sent by node r going through link es. Then the capacity of link e is shared by the flows $f(r,e)$ going through it. Constraints (5) thus represent the flow conservation constraints over the time. It is considered r as the source node of flow f and v is the current node that forwards the received flow following Kirchhoff laws. Two special cases happen: When $v = r$, then the router sends its demand on its ougoing edges. If v is a gateway, it can receive an amount of the flow sent by r, i.e. $y(r, v)$. All the demands have to be sent to the gateways as modeled by constraints (6). Constraints (4) are the link capacity constraints. Total flow on a link cannot exceed its global capacity depending on its activation during the global time period. And (7) claims that the absorbed flow by a router is null if this one is not selected to be a gateway, i.e. binary variable $s(i) = 0$.

4. Wireless mesh network optimizations

By combining the aforesaid constraints (3)–(7), one describes the solution space of all admissible configurations of a given WMN, i.e. the set of configurations of the mesh network satisfying the constraints. On this generic set of constraints, multiple optimizations can be performed, depending both on the objective function and on the variables that are instantiated to constant values of the MILP.

In the following, we present some of the most common optimization problems arising in the settings of WMN and their writing in this model.

4.1. WMN deployment

The gateway placement can be tackled as follows in our model. Within the cross-layer constraints of the routing and scheduling, one can set $s(i)$ as a binary variable determining whether a gateway is installed at node i, and define the two following optimization problems.

4.1.1. Optimal gateway placement problem (GPP)

When traffic demands are known for every router, the gateway placement problem seeks to deploy gateways in the WMN such that all demands are satisfied. Given a set V_g of candidate gateways, we look for the smallest subset of gateways such that the traffic demand is satisfied, that is:

$$\textbf{Objective}: \min \sum_{i\in V_g} s(i)$$

The sum in this objective function can be weighted by a cost associated with each gateway placement.

4.1.2. Fair gateway placement problem (FGPP)

If the number of gateways to be deployed is known, the objective is to place them in order to maximize the throughput in a simplified max–min fairness model. The traffic demand of each router becomes a variable instead of a constant, and we seek to maximize the minimum throughput assigned to a router among all feasible routes and schedules.

$$\textbf{Objective}: \quad \max(\min_{r\in V_r}(d_r))$$
$$\textit{such that}: \quad \text{Eq. (3)–(7) and } \sum_{i\in V_g} s(i) = n$$

Relevance of gateway placement fine tuning Gateway placement problems are very challenging from a combinatorial viewpoint. However, the specific traffic pattern of gathering communications on the gateways induces bottlenecks around the gateways which tightly constrain the available capacity [9]. Indeed, simulation results such as those presented in Section 5.4 assess a very weak impact of the location of the gateways on the capacity of the network. The capacity depends on localized constraints on the placement which are very easy to fit: When two gateways are too close from each other, they actually act as only one because of the interferences which prevent a simultaneous traffic gathering. On the contrary, it is sufficient that each gateway has a large enough neighborhood for the network to provide its maximum capacity.

Therefore, from now on, we focus on WMN capacity optimization while considering that both the mesh routers and the gateways are already deployed. This approach is validated through simulations presented in Section 5.4.

4.2. WMN capacity optimization

Let us be given a WMN and a gateway placement. The $s(i)$ are constants and equal *1* for the set of gateways V_g, *0* otherwise. Evaluating WMN performances is useful to determine the Quality of Service (QoS) that must be guaranteed to the clients. Among the existing performance indicators, we distinguish the network capacity, defined as the amount of traffic the network can carry per unit of time. Being able to provide a high capacity to WMN is deeply investigated since it is known that wireless networks have a scalability issue [5]. Our work aims at providing an upper bound on the achievable capacity of any given WMN topology, independently to any routing or scheduling protocol.

4.2.1. Max-sum flow

A natural way of trying to maximize the network throughput is to maximize the sum of all the flows going through the network:

$$\textbf{Objective}: \max \sum_{r\in V_r} \sum_{e\in \Gamma^+(r)} f(r,e)$$

where $\Gamma^+(r)$ is the set of outgoing links of node r.

The drawback of such an approach is the unfairness induced among the transmitters. Almost all the bandwidth is dedicated to a few radio links [8]. In order to introduce fairness, another objective can be used.

4.2.2. Maximum concurrent flow or fair routing and scheduling problem (FRSP)

The fair routing and scheduling problem introduces fairness in order to guarantee an amount of flow to each node that is proportional to a given demand. It is related to the maximum concurrent flow problem whose goal is to maximize λ, such that $\sum_{e\in\Gamma^+(r)} f(r,e) = \lambda d_r,\ \forall r \in V_r$.

In the following, we introduce a dual viewpoint on maximizing the capacity: Instead of packing the largest possible flow in the available resources, one can minimize the resources needed to cover the requested flow demand. In our settings, this viewpoint is quite relevant since we consider a network at its steady state, hence periodic.

4.3. The shortest schedule or round scheduling problem

When the traffic demand is know and given a priori, the optimization problem of maximizing the network capacity becomes a minimization. When λ units of flow are sent from a router to a gateway each T units of time, routing at maximum rate can be viewed as minimizing T, the length of the time period (because the flow rate is equal to λ/T in steady state). Minimizing the schedule duration to carry the traffic therefore maximizes the network capacity [42].

In our formulation, we introduce a new variable χ_t which indicates if at least one link is activated at time t. The network period length equals the number of used slots, i.e. slots during which at least one link is selected. A bound on this length is fixed, T_{max}, and the set of used slots in the frame $[1,T_{max}]$ is minimized:

$$\textbf{Objective}: \quad \min \sum_{t\leqslant T_{max}} \chi_t$$

$$\text{such that}: \quad \text{Eq. (3)–(7) and } a(t,e) \leqslant \chi_t, \quad \forall e \in E, \quad t \leqslant T_{max}$$

4.4. Formulation limits

The mixed-integer formulations presented for computing WMN optimal configurations all suffer from the number of binary variables of the scheduling constraints, the gateway placement constraints being even more costly. Even for small networks and considering small time periods, the program generates MILPs with a huge number of constraints and integer variables. This combinatorial hardness makes the problem intractable for topologies with more than twenty nodes. Large instances cannot be solved to optimality and only approximate solutions can be obtained [24]. In order to tackle larger topologies, we introduce an approach based on a structural model relaxation.

From now on, we assume a given gateway placement in the network and focus on the joint routing and scheduling problem modeled as a Round Weighting Problem (RWP). It has been introduced by [43] and considers a generic interference model represented by a set of rounds. This problem solves the continuous relaxation of the Round Scheduling Problem (RSP) to satisfy a given demand subject to the multi-access interferences, by allowing an activation duration to each round. Computing the optimal capacity of the network is then a Round Weighting Problem. Besides, this relaxation is effective since, given a round weighting, one can easily build an actual scheduling inducing the same activation duration, hence providing the same capacity.

In the rest of the paper, we present linear formulations that can be solved efficiently using column and cut generation processes. Simulations and duality theory highlight that the WMN capacity is only constrained on some bounded region in the network and confirm our previous assessment on the gateway placement.

5. Relaxation to the round weighting problem

A straightforward continuous relaxation of the scheduling constraints (3) would be irrelevant since the structure of the interference-free scheduling is deeply dependent on the binary form of the $a(t,e)$ variables. For instance, one could set all $a(t,e)$ to 1/2. Constraints (3) would always be satisfied but this feasible solution is useless to build efficient sets of pairwise non interfering transmissions. This is a very generic assessment where disjunctions are expressed by binary variables, like in colouring problems. We therefore express contentions by variables on combinatorial objects, instead of constraints on binary variables.

One can notice that a complete link scheduling is not necessary. Indeed, permuting slots does not change the solution cost since the number of units of flow going through a link is limited by the total number of active slots. The set of activated simultaneous transmissions in an optimal solution is enough to construct an optimal scheduling, assigning a frame to each set of simultaneously activated links in an arbitrary order. Hence the problem can be reduced to a round weighting problem, which considers a generic interference model represented by a set of rounds. This problem solves the continuous relaxation of the shortest schedule problem by allocating slots to each round. By allowing this weighting to be fractional instead of integer, one can keep the fundamental structure of the original problem in an effective relaxed scheme.

5.1. Definition of rounds

Instead of considering the set of interfering transmissions $\mathcal{I}(e)$ for each link $e \in E$, we now focus on the set of compatible transmissions that can be selected at the same time. The interference model stays generic and is now expressed in terms of *rounds*, defined as follows:

Definition 1. (Round) A set $R \subseteq E$ in G is called a round if it contains only transmissions that can be active simultaneously.

In the settings of the binary interference model, that means a set of pairwise non-interfering transmissions: $R \subseteq E,\ \forall e_1, e_2 \in R, e_1 \notin \mathcal{I}(e_2)$ and $e_2 \notin \mathcal{I}(e_1)$. In the settings of the SINR model, that means a set of transmissions such that the SINR Eq. (1) holds at all receiving node.

In the conflict graph modeling a binary interference model, a round actually corresponds to an independent set, or stable set. Recall that an independent set of a graph is a subset of nodes such that there is no link between them. For instance in Fig. 3 representing the binary distance-2 interference model, links A_1 and A_4 are separated by A_2 and A_3. They can thus be in the same round and one can see that $\{A_1,A_4\}$ is an independent set of G_c since the corresponding nodes are not adjacent.

5.2. The path/round formulation

Our work uses the same performance metric as in the RWP [43], saying that we consider sets of pairwise compatible transmissions (the rounds) as input of our problem, and we try to optimize the network performances by selecting rounds in order to route the given traffic with maximum throughput from mesh routers to the gateways.

Each round R can be selected $w(R)$ times during the network period. In the relaxed problem, $w(R)$ becomes the duration of the round selection. Only one round can be selected at a time, hence the sum of the round activation durations equals the period length, which is to be minimized (Eq. (8) of the following mixed-integer linear program). The capacity of a given link is then given by the sum of the activation durations over the rounds containing the link (Eq. (9)). Similarly one can introduce a set of paths instead of the multi-commodity flow constraints on links. Thus the routing problem becomes a selection of the set of paths carrying the flows. Let $\mathcal{P}_r$ be the set of paths going from router r to the gateways and $f(P)$ the amount of flow on path $P \in \mathcal{P}_r$. $\mathcal{P} = \cup_{r \in V_r} \mathcal{P}_r$ denotes the set of all paths, and $\mathcal{R}$ the set of the rounds.

The routing and scheduling problem thus becomes:

$$\min \sum_{R \in \mathcal{R}} w(R) \tag{8}$$

$$\sum_{P \in \mathcal{P}, P \ni e} f(P) \leqslant \sum_{R \in \mathcal{R}, R \ni e} w(R), \quad \forall e \in E \tag{9}$$

$$\sum_{P \in \mathcal{P}_r} f(P) \geqslant d_r, \quad \forall r \in V_r \tag{10}$$

In this formulation, the number of constraints is strongly reduced because flow conservation and round construction constraints are no longer necessary; it is hard coded into the structure of the combinatorial variables. Only capacity constraints (9) and demand constraints (10) are expressed. Conversely, the number of variables is now exponential. Indeed, it exists an exponential number of paths between a source–destination pair in the graph, as well as possible rounds. This formulation cannot be manipulated with the whole set of variables when the size of the network grows.

5.3. The resolution method

Column generation is a prominent approach, based on the duality in linear programming, to deal with the exponential size of variables of a linear program, avoiding the enumeration of all the possible variables. It has already been applied for JRSP and RWP and has proved its efficiency [13,26,44]. Actually, these problems suffer from the exponential number of their variables, i.e. paths and rounds. The set of rounds is a subset of 2^E, it is therefore impossible to enumerate all of them in a graph. Similarly there is an exponential number of paths between two distinct nodes in a graph. The column generation process, described in the next paragraph, allows to quickly compute the optimal solution of the path/round formulation only with a subset of paths and rounds (generating only those that improve the objective function).

Recall that the dual of a linear program is another linear program whose constraints are related to the variables of the former, denoted as the *primal program*. There is a tight binding between a solution of the primal and the values of the variables of its dual. In particular, if a solution of the primal program is sub-optimal, the corresponding values of the dual variables violate at least one dual constraint. Based on this fact, the scheme of the column generation process is the following. First, one solves the primal with a restricted set of active variables (the other variables being set to 0). This instance is thus quickly solved. If there exists a feasible solution of this restricted version of the primal, the current optimal primal solution is obtained, as well as the corresponding values of the variables of the dual problem. The initial sets have to be carefully chosen so that an initial feasible solution exists.

Considering a current solution, optimal on the restricted set of variables, this solution is most probably sub-optimal. The resulting optimal dual variables represent a weight function defined on the objects on which the constraints of the primal program are expressed and might violate a constraint of the dual. They are given as parameters of *pricing subproblems* which seek such a violated constraint of the dual problem as detailed below. If such a constraint exists, it corresponds to a variable (i.e. a column) that can be added to the set of active variable in the primal. The duality of linear programming states that solving again the primal with the updated set of variables could produce a better solution. The process loops until no such column exists. At this point, there is a solution of the primal problem as well as a solution of the dual problem since no constraint is violated. According to the theory of duality, both solutions hence have the same objective value which is the optimum of both the master and the dual problems.

5.3.1. Initialization

As said before, the first step is to carefully chose an initial set of paths and rounds such that a feasible solution of the path/round formulation exists. Let $\mathcal{P}_0$ be the set containing a shortest path from each router to a gateway, and $\mathcal{R}_0 = \{\{e\}, e \in E\}$ be the set of rounds containing each link of the network in a singleton. If one assign the total demand $\sum_{r \in V_r} d_r$ as the weight of each round containing a link in a path from $\mathcal{P}_0$, it ensures that there is enough capacity to route the traffic along the paths from the nodes to the gateways. Considering a current optimal solution so that objective (8) is optimized, the resulting dual variables define weight functions on the nodes and links of the network. We thus investigate the dual formulation in order to define the separation programs generating new paths and rounds.

5.3.2. Dual formulation

In our case, the dual of program (8) involves two constraints corresponding to the path and round variables $f(P)$ and $w(R)$. Introducing the dual variables $y(e), e \in E$ for links associated with constraint (9), and $x_u, u \in V_r$ for mesh routers associated with constraint (10), the dual formulation consists in maximizing a *volume*-like mesure of the network, $\sum_{r \in V_r} d_r x_r$, under the following path length and round weight constraints:

$$\sum_{e \in P} y(e) \geqslant x_{\mathcal{O}(P)}, \quad \forall P \in \mathcal{P} \tag{11}$$

$$1 \geqslant \sum_{e \in R} y(e), \quad \forall R \in \mathcal{R} \tag{12}$$

where $\mathcal{O}(P)$ denotes the source node of path P.

The problem is now to determine if there exists paths and rounds violating these constraints that could improve the value of the objective function of the master program. The column generation process we use for the path/round formulation hence involves two pricing subproblems.

5.3.3. Pricing subproblems

Given the dual link variables, the first subproblem aims at finding a weighted path that violates the corresponding dual constraint (11), that is, a path with associated weight lower than the dual variable associated to its source node:

Definition 2. Given a weight function $y : E \rightarrow \mathbb{R}^+$, the *Minimum Weighted Path Problem* consists in finding a path $P \in \mathcal{P}$ for wich $y(P) = \sum_{e \in P} y(e)$ is minimum.

Shortest path computation from the routers to the gateways always gives either the most violated constraint, hence a good candidate column to be added, or a proof that all constraints hold. If relevant, i.e. if the reduced cost $\sum_{e \in P} y(e) - x_{\mathcal{O}(P)}$ is negative, then the path is added in the current set of variables of the master program. Different algorithms based on multiflow formulations or Dijkstra algorithms can be applied to generate new paths in polynomial time [45]. Specific assumptions in terms of number of hops or any kind of metrics on the generated paths can be specified to constrain the set of eligible paths.

Similarly, with the same link weighting, the second pricing subproblem computes a round with total weight greater than *1*, i.e. violating constraint (12), and such that the total weight of the round is maximized:

Definition 3. Given a weight function $y : E \rightarrow \mathbb{R}^+$, the *maximum weighted round problem* consists in finding a round $R \in \mathcal{R}$ for wich $y(R) = \sum_{e \in R} y(e)$ is maximum.

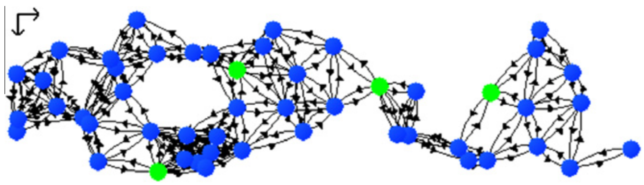


Fig. 4. A generated random graph of 50 nodes and 4 gateways.

Thus, a maximum weighted round generation either gives a good candidate to add to the set of variables, or proves that no such column exists. The second pricing subproblem associated to the round generation can be formulated as an integer linear program with binary variables $y(e)$. If this cost is strictly greater than one, the generated round is added to the set of variables. Constraints of this problem define the structure of the rounds, therefore it fully depends on the interference model chosen. We introduce variables $z_e \in \{0,1\}$ determining if link e is added to the new round or not. For instance, using the generic interference model presented in Section 3, the linear constraints of the second pricing subproblem are the following:

$$z_e + z_{e'} \leqslant 1, \quad \forall e \in E, \quad e' \in \mathcal{I}(e) \tag{13}$$

Recall that $\mathcal{I}(e)$ corresponds to the set of links interfering with e. To consider the *SINR model*, one has to replace constraint (13) by the following ones, where $P_u \in [0, P_{max}]$ is the transmitting power of node u.

$$P_u G(u,v) \geqslant \gamma \cdot (\eta + \sum_{w,x \neq \{u,v\}} P_w G(w,v) z_{(w,x)}) - (1 - z_{(u,v)})\gamma |V| P_{max}, \forall (i,j) \in E \tag{14}$$

$$\sum_{e \in \Gamma(i)} z_e \leqslant 1, \quad \forall i \in V \tag{15}$$

In the case of a binary interference models, computing a maximum weighted independent set of the conflict graph gives a candidate round or certify that no constraint is violated. For instance, for the commonly used binary distance-2 interference model depicted in Fig. 3, the corresponding set of constraints that restricts only one link to be active in any 2-neighborhood is:

$$\sum_{e \in \Gamma(u) \cup \Gamma(v)} z_e \leqslant 1, \quad \forall (u,v) \in E \tag{16}$$

Note that computing maximum weighted independent sets is *NP*-hard in general graphs. However, an experimental observation during simulations is that the weight functions induced by concentration of flow around the gateways, i.e. a flow going indifferently to the set of gateways and not to a specific one, yields instances of maximum weighted independent sets that are quickly solved. Indeed, non zero dual variables are related to constraints that are tight. The dual variables on which the rounds are computed are related to capacity constraints. The non zero values are hence located at the bottlenecks of the network which are only around the gateways if the topology of the network is dense enough.

Special QoS requirements can be easily added in the column generation process, like limiting the number of hops in the paths, or getting more fairness by selecting specific links in the rounds.

5.4. Numerical results

The former MILP, the path/round formulation, and the column generation process have been implemented using the MASCOPT library developed by MASCOTTE team members [46], and solved using ILOG CPLEX solver on a INTEL Core 2 2.4 GHz with 2 Gb of memory.

Table 2
Comparison of the formulations. Time resolution is greatly improved by the path/round formulation.

Net. size	# gtws	Computation time (ms)	
		node/arc RSP	path/round
10	1	4271	448
10	3	4069	512
20	1	107515	1147
20	2	103774	1386
20	5	105485	1055
30	1	713571	6684
50	1	20539940	8513
70	1	...	74516
100	1	...	448912

Tests have been realized on many instances like regular grids (Manhattan like networks) or more general mesh topologies built as follows. A set of n mesh points are deployed on a plane of length *1* and height 1/4, following a Poisson process. A transmission radius is fixed in order to get a connected graph with mean degree $\bar{d} = \max(5, \frac{n}{10})$. Gateways are uniformly and randomly chosen among the nodes. Other nodes becomes routers with a unitary demand to send. We have generated topologies with *10* to 100 nodes. For each topology, we have tested with *1* to $n/\bar{d}$ uniformly spread gateways, where $\bar{d}$ is the mean node degree. In the last case, gateways only communicate with their neighbors, in average. By normalizing distances between the nodes in order to obtain a transmission radius of *1*, we obtain Poisson graphs of density $\frac{n}{10\pi}$. Graphs are then locally dense, but, due to the rectangular property of the plane considered, they stay spread (Fig. 4).

We consider both the binary distance-2 interference model in which a transmission (u,v) competes with all transmissions within a two-hop distance in the transmission graph G, and the Physical model with SINR in wich additive interferences due to simultaneous transmissions are taken into account. In the former case, a round is a subset of links such that two links are at distance at least 3 in G. In the latter case, a round is computed according to the constant parameters given in [30]: Uniform power for mesh nodes $P = 0.002425$ mW, $\eta = 10^{-11}$ mW, $G_{uv} = (d(u,v))^{-3}$, the SINR thresholds $\gamma = \{2.0, 2.8, 7.1, 15.9\}$ and correspondig data rates $D = \{1,2,4,8\}$ depending on the modulation schemes chosen.

5.4.1. Computation time

Computation times of different linear formulations are presented in Table 2. We obtain solution of the path/round formulation for RWP for big topologies in a few seconds, while the former node/arc mixed-integer formulation for the shortest schedule problem RSP cannot handle networks larger than few tens of nodes.

Since the node/arc formulation computes a full slotted link scheduling over time, while the path/round formulation computes a relaxed fractional round weighting, the optimal period length of the latter is a lower bound of the former. As a matter of fact, in any of our test instances, the relaxed period equals the optimal integral one (up to rounding it up). This validate both our approach and the formulation. Preliminary combinatorial results seem to prove this observation up to a small constant in any case.

5.4.2. Capacity vs network size

In a data gathering environment with one gateway, previous works have shown that the per-node capacity is decreasing linearly with the network size [43]. Fig. 5 confirms this phenomenon with several gateways and networks bigger than 20 nodes. We however remark that, in the tested topologies, the computation of new rounds does not take so much time. We think that the particularity

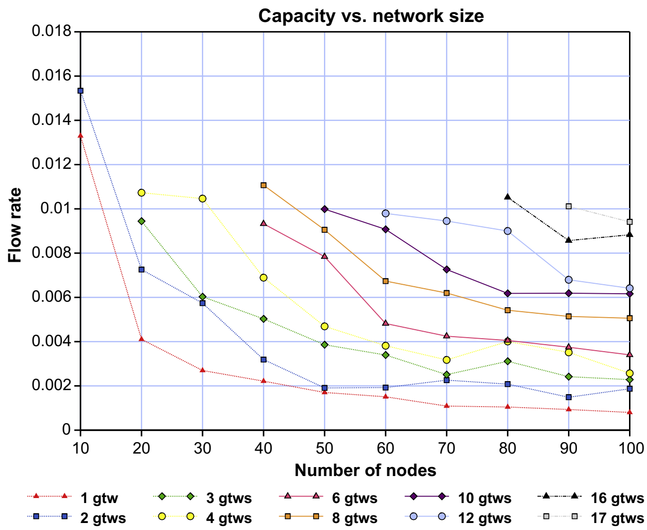


Fig. 5. Capacity evolution vs size of the network.

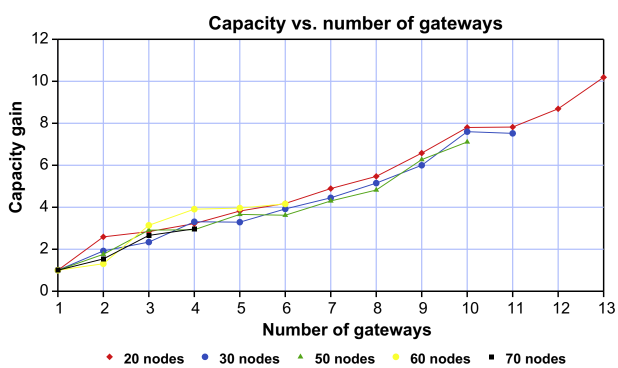


Fig. 6. Almost linear capacity gain when adding a new gateway randomly placed in the network.

of the values of the dual multipliers, as well as some topology characteristics, make the round computation faster than in usual cases. Thus, complexity is not a major issue in the specific case of the weight functions induced by the dual of a concentration flow on the gateways. In particular, we make a deeper study on the values of the dual multipliers and these specific localized characteristics in the next section.

5.4.3. Capacity vs gateway density

Linear dependency between capacity and gateway density corresponds to an obvious upper bound, but interferences between gateways could degrade performances. To study this phenomenon, we have generated different random gateway placements for each topology. Fig. 6 presents the mean gain obtained by adding new gateways. Values are normalized by the rate obtained with one gateway. Results show a linear gain in the number of gateways with a gradient slightly lower than 1. The placement of the gateway is expected to have a strong impact on the network performances. Simulations moderate this assertion. On one hand, when gateways are too close from each other, simulations show that they actually act as only one: interferences on the links around each gateway merge them and prevent a simultaneous traffic gathering. On the other hand, with a correct placement, even non optimized, wireless mesh networks act efficiently. This phenomenon has been identified using the two interference models considered.

Table 3
Capacity depending on the admissible rate.

Net. size	# gtws	Throughput $(D * \sum_{v \in V_r} d_v / OPT)$ given $\{\gamma, D\}$			
		{2.0,1}	{2.8,2}	{7.1,4}	{15.9,8}
20	1	0.56	1.08	2.07	4.16
25	1	1	2	3.56	4.8
25	4	3.6	6.51	9.37	15.24
40	1	0.71	1.79	2.79	3.22
49	1	1	2	3.48	5.19
49	4	3.74	5.86	9.64	14.14
50	1	1.76	3.07	4.15	6.33
50	2	2.48	5.49	7.68	9.3
50	4	3.39	6.77	8.76	10.52

5.4.4. Capacity vs SINR threshold

Table 3 presents the throughput obtained when considering the different SINR thresholds and link data rates. In this case, each network link has a transmission rate and increasing this transmission rate enforces to increase the SINR threshold in order to keep a good transmission quality. If the SINR threshold is more important, then the number of simultaneously transmitting links becomes limited, leading to rounds with fewer density. The optimum round weighting is thus increased due to the limited spatial re-use. However, the number of sent packets associated to each admissible rate is enough to compensate this effect as presented in Table 3 for grid and random wireless mesh networks.

5.4.5. Conjectured requirements for the gateway placement

We conjecture that there exists a minimum interspace between the gateways, that is obviously necessary, but also sufficient, to an efficient traffic flow. This *thickness* is expected to be explicitly computed on regular topologies, and estimated with binary interference models. Preliminary results have shown, on a grid topology with two gateways placed at different distance from each other, from the center to two opposite corners of the grid (see Fig. 7), that the capacity decreases when the gateways are too close either from the border or from each other.

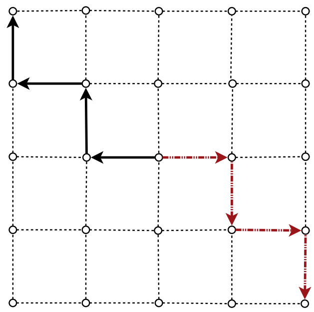
5.5. Further improvements

The path/round formulation presented in Section 5 introduces new assumptions regarding localized properties in wireless mesh networks. An optimization using localized information around the gateways seems to be a necessary condition, or even sufficient, to guarantee the maximum capacity in the network. The analysis of these assumptions are the main topic of the next section.

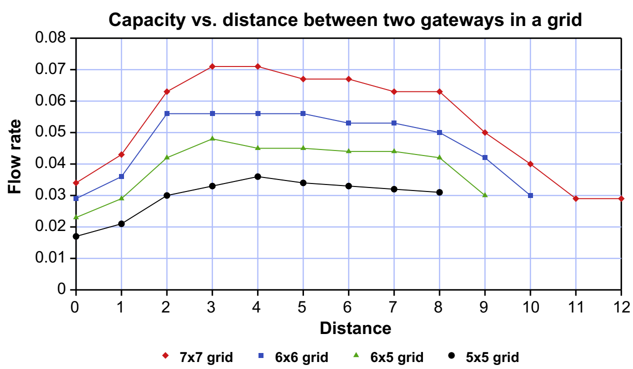
We therefore provide a new cut/round formulation that do not consider the routing and focus on the transport capacity available on the network cuts. A primal/dual algorithm is then derived to solve this formulation using a cross cut and column generation process. Simulations and duality theory highlight that the WMN capacity is only constrained on some bounded region in the network and confirm our previous assessment on the gateway placement. Finally, an improvement presented in Section 6.4.2 makes the practical computation time depends only on the density of nodes and gateways.

6. Optimization using local information

One can remark that, given a round weighting w, the routing problem can be reduced to a maximum flow problem with a single pair (source, destination) by a transformation of the transmission graph G into a modified graph (G', w) depicted in Fig. 8 and defined in the following:



(a) Different placement of 2 gateways in a finite grid.



(b) Capacity vs. the distance between the 2 gateways.

Fig. 7. Study of the gateway placement in a grid network.

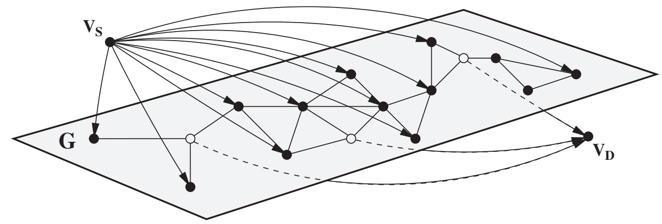


Fig. 8. Associated graph G' extends transmission graph G.

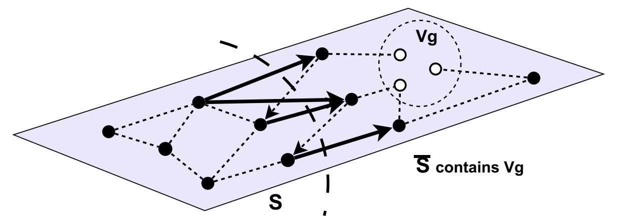


Fig. 9. A cut S in the transmission graph G is a subset of nodes that does not contain the gateways.

Definition 4 (*Associated Graph* (G', w)). Let $G' = (V', E')$ be the graph constructed from transmission graph G and induced link capacity C_w in the following:

- A super source v_S is added with incident links (v_S, r) for every router r of V_r with capacity d_r.
- A sink node v_D is added with incident links (g, v_D) for every gateway g of V_g with infinite capacity.
- Every link e of E has capacity $C_w(e)$.

Thus $V' = V \cup \{v_S, v_D\}$ and $E' = E \cup \{(v_S, r), \forall r \in V_r\} \cup \{(g, v_D), \forall g \in V_g\}$.

This modification allows to use the well-known *max flow/min cut theorem* from graph theory, linear programming and duality theory. Recalling that the maximum flow of a graph equals the capacity of its minimum cut gives us the opportunity to consider only a cut covering problem instead of the routing.

Constraints of the path/round formulation are thus respected if and only if the minimum (v_S, v_D)-cut in G' has a weight greater than $\sum_{v \in V_r} d_v$, the total traffic that has to be carried to the gateways. This leads to a new linear programming formulation described in the next subsection.

6.1. The cut/round formulation

The duality theory of linear programming has shown that one can compute the value of a maximum flow using a minimum cut problem. In the following, we extend this poperty to the RWP and develop a new linear formulation of the problem focusing on the network transport capacity.

In the following, we call $\mathcal{S} \subset 2^V$ the set of cuts of G isolating the gateways: A cut $S \in \mathcal{S}$ is a subset of nodes of V excluding the gateways (see Fig. 9). The border of S, denoted $(S, \overline{S})$, is the set of links connecting a node of S to a node of the complementary set $\overline{S} = V \setminus S$. We thus define the cut traffic $d_S = \sum_{v \in S} d_v$ to be the traffic that must cross its border. In the same way, a cut capacity $C_w(S)$ is induced by the weight function w on the links and is defined by $C_w(S) = \sum_{e \in (S,\overline{S})} C_w(e)$. If we recall the definition of the induced capacity $C_w(e)$, one can obtain the following relation: $C_w(S) = \sum_{R \in \mathcal{R}} \delta(R, S) w(R)$, where $\delta(R, S) = |R \cap (S, \overline{S})|$ represents how many times a round R covers S's border.

From that point, ensuring a sufficient network capacity to carry the flow consists in covering the network cuts of $\mathcal{S}$ by the rounds. Then, the max flow/min cut theorem ensures an optimal solution to correspond to a feasible routing in the network satisfying router demands (cf theorem and proof in the next section). We now derive from these statements a formulation of the cut covering problem:

$$\min \sum_{R \in \mathcal{R}} w(R) \quad (17)$$

$$\sum_{R \in \mathcal{R}} \delta(R, S) w(R) \geqslant d_S, \quad \forall S \in \mathcal{S} \quad (18)$$

6.2. Equivalence proof

Linear program (17)-(18) computes an optimal round weighting such that every cut capacity is greater than the traffic of the cut that has to cross its border. The following Theorem 1 ensures that the induced cut capacities are necessary and sufficient to validate the existence of a feasible routing in the network.

Theorem 1. *Formulations path/round and cut/round compute equivalent optimal round weightings.*

Proof. Let w_1 be a feasible weighting of the path/round program, and Φ a feasible flow in G according to the link capacities induced by w_1. Since Φ is feasible, we know that $\sum_{P \in \mathcal{P}_v} \Phi(P) = d_v$ for each router v. Thus, if we pick a cut S in G that isolates the gateways, flow conservation on each path P of P_r going from r to a gateway ensures that P contains a link of the border $(S, \overline{S})$. This gives:

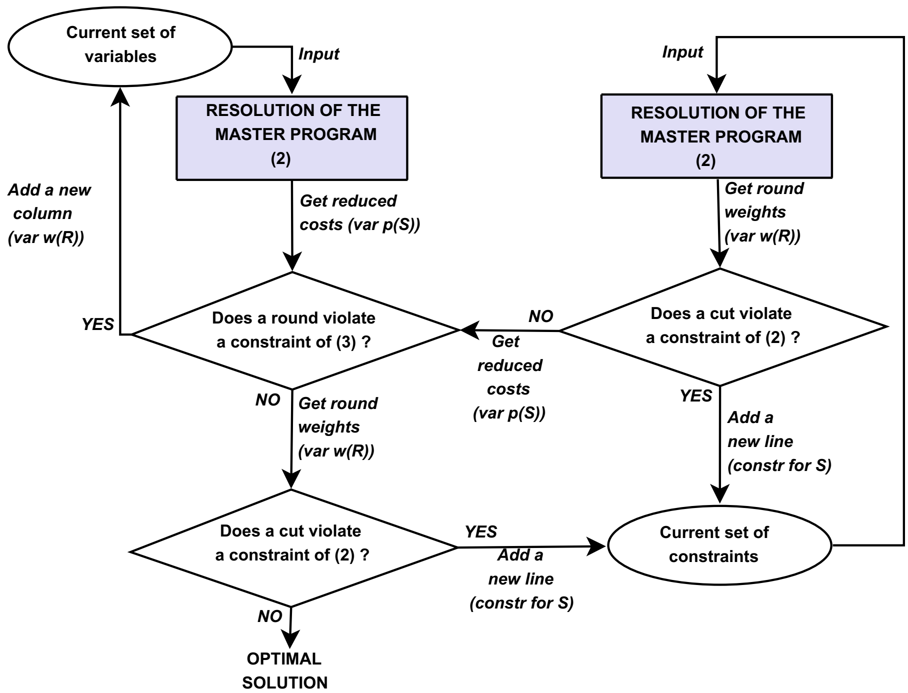


Fig. 10. The cross cut and column generation process.

$$C_{w_1}(S) = \sum_{e\in(S,\bar{S})} C_{w_1}(e) \geqslant \sum_{e\in(S,\bar{S})} \sum_{P\in\mathcal{P},e\in P} \Phi(P) \geqslant \sum_{v\in S} d_v$$

using satisfied capacity constraints (9). By injecting w_1 in the cut/round program, we obtain a feasible solution since

$$\forall S, \sum_{R\in\mathcal{R}} \delta(R,S)w_1(R) = C_{w_1}(S) \geqslant d_S.$$

That is, the cut capacity is large enough to allow its traffic to cross its border.

In particular, an optimal solution of the first program is an upper bound of the solutions of the second program.

Conversely, let w_2 be a feasible solution of the cut/round program. Then a minimum (v_S, v_D)-cut S^* (according to its capacity) in the graph (G', w_2) can be linked to a unique cut S in G such that $S^* = \{v_S\} \cup S$ and $(S^*, \overline{S^*}) = \{(v_S, v), v \in \bar{S}\} \cup (S, \bar{S})$.

One can remark that the gateways cannot be in S since there would be some links (g, v_D) in S^*. As S^* is a minimum cut, it cannot contain links with infinite capacity.S^* capacity is now expressed by:

$$C_{w_2}(S^*) = \sum_{v\in\bar{S}} C_{w_2}((v_S, v)) + \sum_{e\in(S,\bar{S})} C_{w_2}(e).$$

And thus $C_{w_2}(S^*) = \sum_{v\in\bar{S}} d_v + C_{w_2}(S)$.

The corresponding constraint (18) thus ensures that $C_{w_2}(S) \geqslant d_S = \sum_{v\in S} d_v$. Then $C_{w_2}(S^*) \geqslant \sum_{v\in V_r} d_v$ and the max flow/min cut theorem guarantee the presence of a feasible flow in (G', w_2) with value $\sum_{v\in V_r} d_v$, that is, a feasible solution of the path/round program. In particular, optimal solutions of the cut/round program upper bound the solutions of the path/round program, which completes our proof. □

From a round weighting solution w, one can construct the associated graph (G', w) and compute a maximum flow of value $\sum_{v\in V_r} d_v$ from the super source v_S to the sink node v_D. The Ford and Fulkerson algorithm, or the push/relabel algorithm introduced by Goldberg and Tarjan [47] allows to find the set of paths from each router to the gateways in polynomial time.

6.3. Cross cut and column generation process

In this cut/round formulation, we deal not only with an exponential number of variables (one for each round), but also with an exponential number of constraints (one for each cut). The corresponding method to avoid the complete enumeration of the constraints is the cut generation. To solve the new cut/round formulation, we have to combine cross cut and column generation. This leads to a primal–dual algorithm described in the following.

The process starts as previously with a small subset of variables and constraints. From an optimal solution of the master problem (17)-(18) with restricted sets of rounds and cuts, the process seeks to generate new rows (cuts) or columns (rounds) to add to the formulation in order to improve the solution. A new cut is found when a constraint is violated, and a new column corresponds to a variable with value zero that we want to change, i.e. a variable violating a dual constraint (Fig. 10).

In the cut/round formulation, a constraint is violated if a cut capacity is too weak to support its own outgoing traffic, i.e. if $\sum_{R\in\mathcal{R}} \delta(R,S)w(R) < d_S$ for a given solution w. This inequality can be rewritten as $\sum_{e\in(S,\bar{S})} C_w(e) - \sum_{v\in S} d_v < 0$. As done in the previous section, let us project this equation on the associated graph (G', w) with $S^* = \{v_S\} \cup S$ and $(S^*, \overline{S^*}) = \{(v_S, v), v \in \bar{S}\} \cup (S, \bar{S})$. The capacity of the edges (v_s, v) is $C_w(v_s, v) = d_v$.

The equation becomes $\sum_{e\in(S*,\overline{S*})} C_w(e) - \sum_{v\in V} d_v < 0$, i.e. $\sum_{e\in(S*,\overline{S*})} C_w(e) < \sum_{v\in V} d_v$. Row generation is hence computed by a minimum cut algorithm in the weighted graph (G', w): if the capacity of the minimum cut of G' is greater than its traffic, then all constraints of the primal holds. Otherwise, the minimum cut algorithm gives a candidate to add to the current set of constraints.

The *separation program* for the cut generation aim to compute new lines that could improve the solution of the problem. For sake

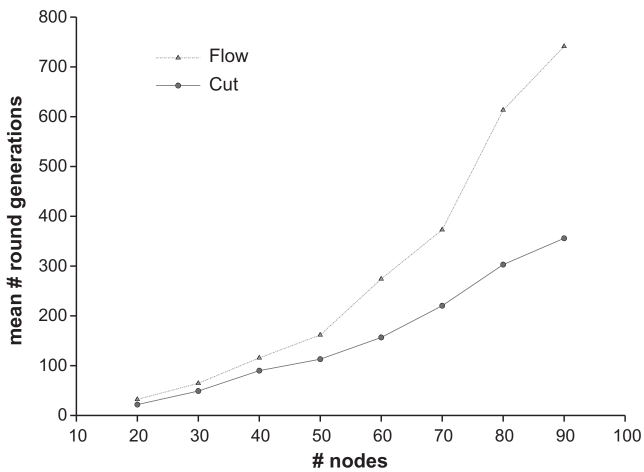


Fig. 11. Number of rounds generated in the optimization of cut and flow formulations.

of simplicity we choose to compute an integer linear program whose formulation is expressed on the transmission graph G even though is computes the minimal cut of G'. Given the actual optimal weights w, $y(e)$ is a binary variable saying if link e is in the cut's border, and $x(v)$ is another binary variable representing the node's selection to be in the cut.

$$\min \sum_{e \in E} C_w(e) y(e) - \sum_{v \in V_r} d_v x(v) \tag{19}$$

$$x(u) - x(v) \leqslant y((u,v)), \quad \forall (u,v) \in E \tag{20}$$

$$x(v) = 0 \quad \forall v \in V_g \tag{21}$$

In order to find violated constraints of the cut/round program, the objective seeks to minimize the difference between the capacity of the cut, i.e. the sum of the capacity of the links in its border, and the traffic of the cut, i.e. the sum of the traffic of each node in the cut. Constraints say that the gateways cannot be selected in the cut, and that a link has to be counted in the border if its source node is in the cut and its destination node is not.

Surprisingly, this ILP runs fast and gives the optimal solution quasi instantly. We will see later that simulations give a hint that complexity is not a major issue in the specific case of the round weighting.

The Linear Program presented below is the dual formulation of the cut/round formulation. It corresponds to a round packing by cuts balanced with $p(S)$. Each round capacity is less than or equal to 1 and the objective is to maximize a profit based on cut traffic. In other words, a constraint is not satisfied if a round has an induced cost greater than *1*. Generate a column of the master program is done by identifying a violated constraint of this dual program when $p(S)$ is given by the reduced costs of the current solution.

$$\max \sum_{S \in \mathcal{S}} p(S) d_S \tag{22}$$

$$\sum_{S \in \mathcal{S}} \delta(R,S) p(S) \leqslant 1, \quad \forall R \in \mathcal{R} \tag{23}$$

The pricing subproblem is the same as in the path/ round generation: Maximize the round weight, i.e. the sum of the weights of the links in the round, given the actual induced costs p: $\max \sum_{e \in E} (\sum_{S \in \mathcal{S}, e \in (S,\bar{S})} p(S)) y(e)$.

The cross cut and column generation process is translated into a primal–dual algorithm described in the Algorithm 1, that works as follows. The algorithm starts with the simplest cut of the transmission graph G isolating the gateways, i.e. the set $\mathcal{S}_0 = \{S_0 = V_r\}$ containing all the mesh routers, and a set of rounds containing the singletons $\mathcal{R}_0 = \{\{e\}, e \in E\}$. The cut/round program computes the cut covering by the rounds, leading to a current local optimal solution. Also, a feasible solution always exists with $\mathcal{S}_0$ and $\mathcal{R}_0$ since all links (r,g) between each router $r \in V_r$ and a gateway $g \in V_g$ forming the border of S_0 can be activated d_r times. Then one checks if all the cuts of G are covered by the rounds, otherwise the row generation is processed to add the non-covered cuts to the set of constraints. This process is repeated until no such rounds and cuts exist. Then, the separation/optimization theorem [48] ensures that we have found the optimal solution.

Algorithm 1. Primal–Dual Algorithm for RWP in WMNs.

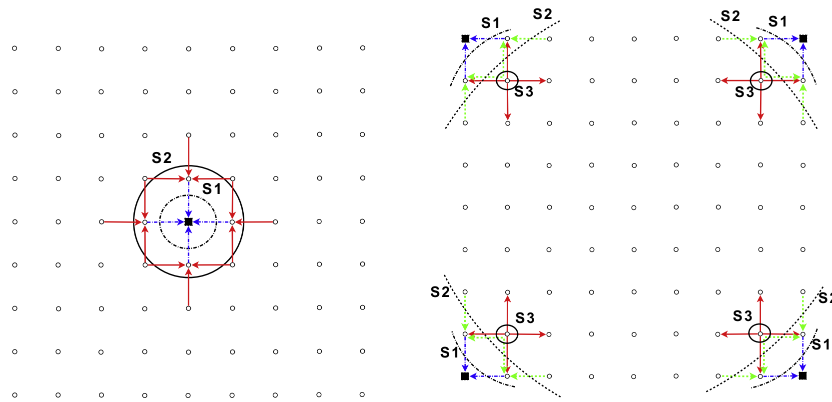
Require network graph G
Ensure a round weighting in G
  $\mathcal{S} \leftarrow \{S_0 = V_r\}$
  $\mathcal{R} \leftarrow \{\{e\}, \forall e \in E\}$
  Solve formulation (17) and (18)
  $\mathcal{R}_{new} \leftarrow$ Get violating rounds:
    $\{R,\ s.t.\ 1 < \sum_{S \in \mathcal{S}} \delta(R,S) p(S)\}$
  $\mathcal{S}_{new} \leftarrow$ Get violating cuts:
    $\{S,\ s.t.\ d_S > \sum_{R \in \mathcal{R}} \delta(R,S) w(R)\}$
  while $(\mathcal{R}_{new} \neq \emptyset)$ || $(\mathcal{S}_{new} \neq \emptyset)$ **do**
   while $(\mathcal{R}_{new} \neq \emptyset)$ **do**
    $\mathcal{R} \leftarrow \mathcal{R} \cup \{\mathcal{R}_{new}\}$
    Solve formulation (17) and (18)
    $\mathcal{R}_{new} \leftarrow$ Get violating rounds
  end while
  $\mathcal{S}_{new} \leftarrow$ Get violating cuts
  while $(\mathcal{S}_{new} \neq \emptyset)$ **do**
    $\mathcal{S} \leftarrow \mathcal{S} \cup \{\mathcal{S}_{new}\}$
    Solve formulation (17) and (18)
    $\mathcal{S}_{new} \leftarrow$ Get violating cuts
  endwhile
  $\mathcal{R}_{new} \leftarrow$ Get violating rounds
end while
return w.

6.4. Numerical results

This approach has also been validated through extensive simulations using the MASCOPT library and ILOG/IBM CPLEX solver on a INTEL Core 2 2.4 GHz with 2 Gb of memory. The simulation run validate our approach as it confirms existing results on the wireless network capacity.

The cut and column generation primal–dual algorithm quickly solves our round weighting instances: From tenth of seconds to a few minutes for topologies of size 10 to 80 nodes, and even for topologies with more than 100 nodes with the binary interference model. On one hand, it allows to solve large-scale instances to optimality. On the other hand, the computational time is roughly the same as the path/round formulation with flows [44]: Sub-linear in the network size and linear in the gateways density.

Moreover, one can see in Fig. 11 that the number of generated rounds is decreased in comparison to the existing formulation with flows. This is interesting when considering more sophisticated interference models like SINR power-based models. This is also better since the pricing subproblem generating new rounds is an ILP related to the *maximum independent set problem* which is known to be *NP*-hard in general graphs, even if in practical the round generation is very fast. A more detailed look on the results



(a) 1 gateway at the center.
(b) 4 gateways at corners.

Fig. 12. A small number of cuts is enough to find the optimal solution of grid networks.

show that generating the rounds is sub-linear in the network size, but almost linear in the gateways density.

In Section 5.4, we have conjectured that there exists a minimum interspace between the gateways, that is obviously necessary, but also sufficient, to an efficient traffic flow. We thus use the structural properties of the cut formulation to empirically highlight this thickness, which is also denoted *bottleneck area* and pointed out in other researches [49,50].

6.4.1. Highlighting the bottleneck area

In grid topologies, previous researches have shown exact bounds of RWP in the case of one gateway located at the center or at the corner of a grid [49]. In the proof, authors use the primal and dual multiplier values to show that only the 2-neighborhood of the gateway matters.

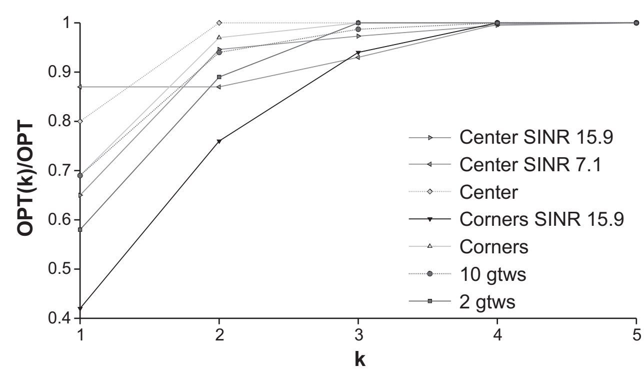
In our study, we say that a cut is active if it has a strictly positive reduced cost, following that its corresponding constraint in the program Eq. (2) is tight. A first result with our method on the grid show that all the active cuts are located in the 2-neighborhood of the gateways (Fig. 12) with the distance-2 interference model. Our results really highlight the area bounding the solution, confirming the work of [49] and generalizing it on other topologies and with several gateways.

Since the SINR model is not a symmetric interference model as the distance-2 model, the structure of the rounds generated cannot be the same. Consequently, in grid topologies with one gateway located in the centre, when $\gamma = 2.0$ or 2.8, it is possible to simultaneously activate a link incident to the gateway with another link incident to a neighbour of the gateway. The traffic can therefore be continuously sent to the gateway without loss, leading to an optimal value equals to the total network demand, and only one active cut in the optimal solution (the initial cut $C_0 = \{V_r\}$). Increasing the SINR threshold has the effect of reducing the density of the rounds, as already discussed in Section 5.4. Therefore, we obtain two active cuts (as depicted in Fig. 12(a)) when SINR threshold is 7.1, when the rounds density is roughly the same as the one with the distance-2 interference model.

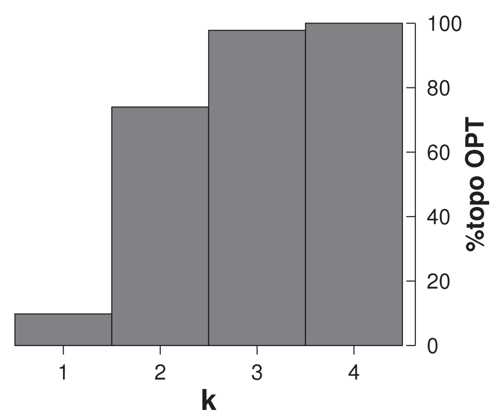
6.4.2. Improving the performances

In order to identify this bottleneck area, we do the following process. Given an integer k as input, the algorithm only changes in the separation problems of the cross cut and column generation process in which new cuts and rounds are generated. We forces the program to only compute rounds whose links are located in the k-neighborhood of a gateway. Similarly, cuts must have the following property: Their border, i.e. the set of links going from a vertex in the cut to a vertex outside it, is in the k-neighborhood of the gateways.

Depending on the value of k, the optimum obtained is lower or equal to the optimal one found on the entire graph: $OPT_1 \leqslant OPT_2 \leqslant \cdots \leqslant OPT_k \leqslant \cdots \leqslant OPT$. Indeed, the set of cuts considered, i.e. the constraints of the problem, is limited. Therefore, either we have enough constraints and we find the same result as in the original problem, or the problem is sub-constrained and the optimal is less than the optimal solution of the original minimization problem.



(a) Optimality gap in function of k for grid topologies for different gateway placements.



(b) Percentage of networks that reach the optimal value in function of k.

Fig. 13. Highlighting the contention area.

Table 4
Comparison between the computation time (ms) of the different formulations

Topology	Net. size	# gtws	node/arc	path/round	cut/round	cut/round improved
Grid	5×5	1	4359879	1205	2728	452
Grid	7×7	1	31045708	9512	3210	641
Random	50	1	20539940	38756	49413	2744
Random	100	1	...	448912	166722	8505
Random	100	9	...	7512539	2146999	619959
Grid	15×15	1	...	56248	131919	2043
Grid	15×15	4	...	316213	423332	2259
Grid	$\mathbf{n} \times n$	1 center Or 4 corners	–	–	–	A few seconds
Random	$\mathbf{n}$	$\frac{\mathbf{n}}{\mathbf{d}}$	–	–	–	A minute/gtw

We solved the problem to optimality for $k = 1,\ldots,5$ on hundreds of topologies, both using the distance-2 and the SINR interference models. Some results are depicted in Fig. 13(a) for grid topologies. More generally, Fig. 13(b) presents the percentage of networks solved to optimality in function of the value of k with the binary interference model. One can remark that optimal solutions are mostly solved when $k = 2$ or 3, and all optimal solutions have been reached when $k = 4$. Actually, the cases when $OPT_1 = OPT$ happen when the number of gateways is big in comparison to the network size, e.g. the 1-neighborhood of the gateways contains all the nodes. On the contrary, networks where $OPT_3 < OPT$ have a size bigger than 70 nodes in which the node connectivity is really weak around the gateways.

7. Conclusion

In this work, we have presented a cross-layer method for optimization problems arising on Internet-providing wireless mesh networks. We have developed exact formulations using integer linear programming and their relaxation that are solved using sophisticated processes like column and cut generation. This approach allows to compute solutions for large scale networks and study the capacity offered to clients upon different parameters. The efficiency of the different formulations is depicted in Table 4 where we present the computation time of each method to compute the optimal solution of the joint routing and scheduling problem in WMNs.

Primary simulations highlight the fact that tightly optimizing the mesh gateway placement seems to be irrelevant. A critical area centered at the gateways beyond which the problem can be roughly solved. These remarks can have an impact on the capacity of wireless mesh networks as much as on the complexity of their design.

Then, we have presented a new formulation for the round weighting problem that forgets the routing to focus on the capacity available on the network cuts. This yields a primal–dual algorithm combining cross column and cut generation that uses a joint cut and column generation process to deal with large scale instances. This cut/round formulation has been validated from the path/round formulation of RWP by proving the equality of the optimal solutions. Moreover, we conjecture that the proposed algorithm generates a number of cuts and rounds which is polynomial in the number of nodes. Proving that result would be a major step toward stating a fixed parameter tractable complexity of the optimization of the network, which means polynomial in the number of nodes but exponential in a (hopefully small) parameter such as the density of the nodes or the mean distance between admissible concurrent transmissions.

An asset of the cut/round formulation is to point out a bounded region, a ‘‘bottleneck’’ of the network, that is enough to optimize in order to get the optimal solution of the round weighting problem of the whole network. The location of these bottlenecks around the gateways are due to several factors. Obviously, the network connectivity has to be large enough. A power control functionality may also have an impact on these gateways. On the other hand, the major cause of these bottlenecks are the concentration of the traffic on the gateways and the importance of distance in the interference models. As a future work we would like to analyse the impact of power assignment in wireless mesh networks on the size and location of the bottlenecks. This would be interesting in order to allow a better link density in the region of the network where more capacity is needed, namely the bottleneck region.

This approach is very useful for practical use. Actually, one can deploy a network that is carefully optimized in a bounded area containing the gateways. In this area, a conflict-free link scheduling is carried out optimally by each gateway for its neighborhood. Then, one can combine approximation algorithms like distributed routing algorithms outside the area that spread the traffic among the mesh routers and bring it correctly to the contention area without degrading the achieved capacity.

Acknowledgments

C. Molle-Caillouet PhD has been funded by the French Defense Agency (DGA), France. This work has been partially funded by european project IST/FET AEOLUS, ANR-JC OSERA, ANR VERSO ECOSCELLS, and ARC CARMA.

References

[1] I. Akyildiz, X. Wang, W. Wang, Wireless mesh networks: a survey, Computer Networks 47 (4) (2005) 445–487.
[2] K. Jain, J. Padhye, V. Padhamanabhan, L. Qiu, Impact of interference on multi-hop wireless network performance, in: ACM MobiCom, 2003, pp. 66–80.
[3] M. Kodialam, T. Nandagopal, On the capacity region of multi-radio multi-channel wireless mesh networks, in: IEEE WiMesh, 2005.
[4] W.J. Cook, W.H. Cunningham, W.R. Pulleyblank, A. Schrijver, Combinatorial Optimization, John Wiley & Sons Inc., New York, NY, USA, 1998.
[5] P. Gupta, P. Kumar, The capacity of wireless networks, IEEE Transactions on Information Theory 46 (2) (2000) 388–404.
[6] O. Dousse, M. Franceschetti, D. Tse, P. Thiran, Closing the gap in the capacity of random wireless networks, in: IEEE ISIT, 2004.
[7] Y.J. Hwang, S.-L. Kim, The capacity of random wireless networks, IEEE Transactions on Wireless Communications 7 (12) (2008) 4968–4975.
[8] H. Rivano, F. Theoleyre, F. Valois, A framework for the capacity evaluation of multihop wireless networks, AdHoc and Sensor Wireless networks 9 (3-4) (2010) 139–162.
[9] J. Jun, M. Sichitiu, The nominal capacity of wireless mesh networks, IEEE Wireless Communications 10 (5) (2003) 8–14.
[10] P. Zhou, X. Wang, R. Rao, Asymptotic capacity of infrastructure wireless mesh networks, IEEE Transactions on Mobile Computing 7 (8) (2008) 1011–1024.
[11] H. Liu, B. Zhao, Optimal scheduling for link assignment in traffic-sensitive stdma wireless ad-hoc networks, in: ICCNMC, Vol. 3619 of LNCS, 2005, pp. 218–228.
[12] S. Mukherjee, H. Viswanathan, Throughput-range tradeoff of wireless mesh backhaul networks, IEEE Journal on Selected Areas in Communications 24 (3) (2006) 593–602.
[13] J. Zhang, H. Wu, Q. Zhang, B. Li, Joint routing and scheduling in multi-radio multi-channel multi-hop wireless networks, in: IEEE BROADNETS, 2005, pp. 678–687.

[14] P. Bjorklund, P. Varbrand, D. Yuan, Resource optimization of spatial tdma in ad hoc radio networks: a column generation approach, in: IEEE Infocom, 2003.
[15] L. Xiao, M. Johansson, S.P. Boyd, Simultaneous routing and resource allocation via dual decomposition, IEEE Transactions on Communications 52 (7) (2004) 1136–1144.
[16] J. Luo, A. Girard, C. Rosenberg, Efficient algorithms to solve a class of resource allocation problems in large wireless networks, in: WiOpt, 2009.
[17] A. Iyer, C. Rosenberg, A. Karnik, What is the right model for wireless channel interference?, IEEE Transactions on Wireless Communications 8 (5) (2009) 2662–2671
[18] D.D. Couto, D. Aguayo, B. Chambers, R. Morris, Performance of multihop wireless networks: Shortest path in not enough, ACM SIGCOMM Computer Communication Review 33 (2003) 83–88.
[19] F. Uddin, C. Rosenberg, W. Zhuang, A. Girard, Joint configuration of routing and medium access parameters in wireless networks, in: IEEE Globecom, 2009, pp. 1–8.
[20] C. Hsu, P. Hung, J. Sheu, Scheduling of broadcasts in multihop wireless networks, in: The handbook of ad-hoc wireless networks, 2003, pp. 483–495.
[21] J.C. Bermond, N. Nisse, P. Reyes, H. Rivano, Minimum delay data gathering in radio networks, in: Ad-Hoc-Now, LNCS, Springer, vol. 5793, 2009, pp. 69–82.
[22] J.-C. Bermond, M.-L. Yu, Optimal gathering algorithms in multi-hop radio tree networks with interferences, Ad-Hoc and Sensor Wireless Networks vol. 9, pp. 1–2.
[23] V. Bonifaci, P. Korteweg, A. Marchetti-Spaccamela, L. Stougie, An approximation algorithm for the wireless gathering problem, in: SWAT, 2006, pp. 328–338.
[24] C. Gomes, C. Molle, P. Reyes, Optimal design of wireless mesh networks, in: JDIR, 2008, pp. 67–76.
[25] M. Johansson, L. Xiao, Cross-layer optimization of wireless networks using nonlinear column generation, IEEE Transactions on Wireless Communications 5 (2) (2006) 435–445.
[26] G. Carello, I. Filippini, S. Gualandi, F. Malucelli, Scheduling and routing in wireless multi-hop networks by column generation, in: INOC, 2007.
[27] W.-H. Tam, Y.-C. Tseng, Joint multi-channel link layer and multi-path routing design for wireless mesh networks, in: IEEE Infocom, 2007, pp. 2081–2089.
[28] S. Lakshmanan, K. Sundaresan, R. Sivakumar, On multi-gateway association in wireless mesh networks, in: IEEE WiMesh, 2006, pp. 135–137.
[29] A. Capone, G. Carello, I. Filippini, S. Gualandi, F. Malucelli, Solving a resource allocation problem wireless mesh networks: A comparison between cp-based and a classical column generation, Networks 55 (3) (2010) 221–233.
[30] A. Capone, G. Carello, I. Filippini, S. Gualandi, F. Malucelli, Routing, scheduling and channel assignment in wireless mesh networks: optimization models and algorithms, Ad-Hoc Networks 8 (6) (2010) 545–563.
[31] E. Amaldi, A. Capone, M. Cesana, I. Filippini, F. Malucelli, Optimization models and methods for planning wireless mesh networks, Computer Networks 52 (11) (2008) 2159–2171.
[32] K. Aardal, D. Schmoys, E. Tardos, Approximation algorithms for facility location problems, in: 29th Annual ACM Symposium on Theory of Computing, 1997, pp. 265–274.
[33] W. Youssef, M. Younis, Intelligent gateways placement for reduced data latency in wireless sensor networks, in: IEEE ICC, 2007, pp. 3805–3810.
[34] P. Li, X. Huang, Y. Fang, P. Lin, Optimal placement of gateways in vehicular networks, IEEE Transactions on Vehicular Technology 56 (6) (2007) 3421–3430.
[35] R. Prasad, H. Wu, Gateway deployment optimization in cellular wi-fi mesh networks, Journal of Networks 1 (3) (2006) 31–39.
[36] F. Li, Y. Wang, X.-Y. Li, A. Nusairat, Y. Wu, Gateway placement for throughput optimization in wireless mesh networks, Mobile Networks and Applications 13 (1-2) (2008) 198–211.
[37] Y. Drabu, H. Peyravi, Gateway placement with qos constraints in wireless mesh networks, in: ICN, 2008, pp. 46–51.
[38] S.N. Muthaiah, C.P. Rosenberg, Single gateway placement in wireless mesh networks, in: IEEE ISCN, 2008.
[39] R.K. Kave, R. Khalili, K. Salamatian, Evaluation of packet error rate in wireless networks (2004). doi:10.1.1.58.8941.
[40] P. Wang, S. Bohacek, Communication models for throughput optimization in mesh networks, in: PE-WASUN, 2008, pp. 80–87.
[41] A. Kumar, M. Marathe, S. Parthasarathy, A. Srinivasan, Algorithmic aspects of capacity in wireless networks, in: ACM SIGMETRICS, vol. 33, 2005, pp. 133–144.
[42] S. Max, E. Weiss, G.R. Hiertz, B. Walke, Capacity bounds of deployment concepts for wireless mesh networks, Performance Evaluation 66 (3-5) (2009) 272–286.
[43] R. Klasing, N. Morales, S. Pérennes, On the complexity of bandwidth allocation in radio networks, Theoretical Computer Science 406 (3) (2008) 225–239.
[44] C. Molle, F. Peix, H. Rivano, An optimization framework for the joint routing and scheduling in wireless mesh networks, in: IEEE PIMRC, 2008.
[45] C. Molle, F. Peix, H. Rivano, Cross-layer design for wireless mesh networks using column generation, Technical Report 6448 INRIA, 2007.
[46] J.-F. Lalande, M. Syska, Y. Verhoeven, Mascopt – a network optimization library: graph manipulation, Technical Report 0293, INRIA, 2004.
[47] A. Goldberg, R. Tarjan, A new approach to the maximum flow problem, in: ACM STOC, ACM, New York, NY, USA, 1986, pp. 136–146. doi: http://doi.acm.org/10.1145/12130.12144..
[48] M. Grotschel, L. Lovasz, A. Schrijver, The ellipsoid method and its consequences in combinatorial optimization, Combinatorica 1 (2) (1981) 169–197.
[49] C. Gomes, S. Pérennes, P. Reyes, H. Rivano, Bandwidth allocation in radio grid networks, in: AlgoTel, Saint-Malo, France, 2008, pp. 41–44.
[50] P. Reyes, Data gathering in radio networks, Ph.D. thesis, Université de Nice-Sophia Antipolis (UNS) August 2009.