

# Network Dynamics: Network Calculus

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# Network calculus

## Model

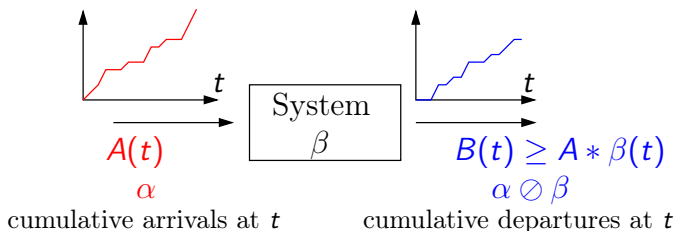
In a network, network elements, such as servers and data flows are modeled by functions in the  $(\min, \text{plus})$  algebra.

- the propagation of the characteristics of the flows is made by algebraic computations
- deterministic performance bounds can be obtained by simple operations.

## Challenges

- 1 The bounds are over-optimistic
- 2 Several related models that induce some confusion.
- 3 Applications: originally, for the analysis of communication networks, now, mainly for embedded networks

# Network calculus



- $A : \mathbb{R}_+ \rightarrow \mathbb{R}_{\min+}$ : process of the cumulative arrivals, non-decreasing function
- $B : \mathbb{R}_+ \rightarrow \mathbb{R}_{\min+}$ : process of the cumulative departures, non-decreasing function
- Causality constraint :  $A \geq B$

# Achievements

## Model

- Comparison of the models
- Packetisation operator

## Algorithms

- Efficient algorithms for operations on  $(\min, \text{plus})$  functions
- Implementation: scibal toolbox,  $(\min, \text{plus})$  console

## Performances

- Tight performance bounds in feed-forward networks
- Improve existing bounds for some service policies

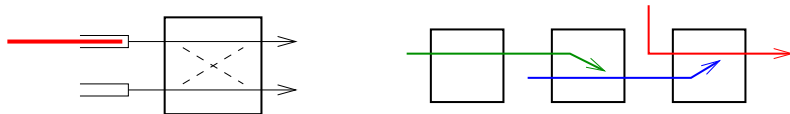
# ZOOM 1: packetization

[Bruno Gaujal (INRIA, Mescal) & Nadir Farhi (IFFSTAR)]

- **Arrival curves:**  $\underline{\alpha}(t - s) \leq A(t) - A(s) \leq \bar{\alpha}(t - s)$  bound the number of arrivals between time  $s$  and  $t$ .
- **Packet curve:**  $\underline{\pi}(y - x) \leq P(y) - P(x) \leq \bar{\pi}(y - x)$  bound the number of packets contained between the  $x$ -th and  $y$ -th bit of data that arrived for the flow.

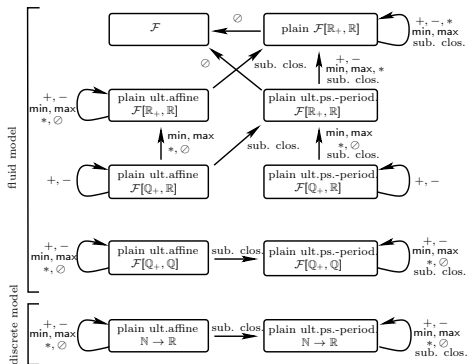
Improves the bounds for the (generalized) Round-Robin policy.

**Target application:** wormhole routing, network-on-chip.



# ZOOM 2: algorithms and implementation

[Éric Thierry (ENS Lyon), Nicolas Navet et al. (INRIA, RT@W), Marc Boyer (ONERA)]

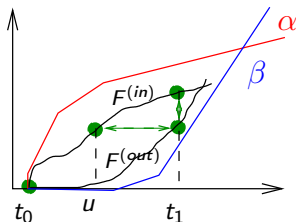


## Implementation

- 1 Scilab toolbox (COINC toolbox)
- 2 (Min,plus) console and PEGASE prototype - can be used to validate AFDX network (used in large civil airplanes).

## ZOOM 3: performances

[Éric Thierry (ENS Lyon), Aurore Junier (INRIA, DistribCom)]

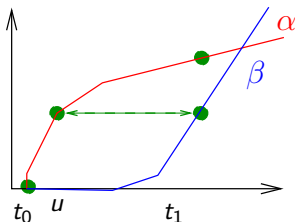


### Objective

- Maximum delay: Maximize  $t_1 - u$  with  $F^{(in)}(u) \geq F^{(out)}(t_1)$
- Maximum backlog: Maximize  $F^{(in)}(t_1) - F^{(out)}(t_1)$ .

## ZOOM 3: performances

[Éric Thierry (ENS Lyon), Aurore Junier (INRIA, DistribCom)]



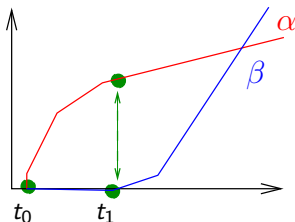
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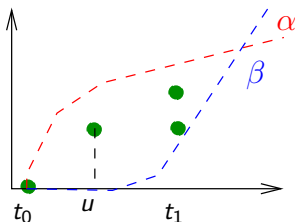


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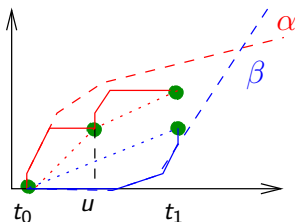


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- Arbitrarily multiplexing:
  - (min,plus) algebra  $\rightarrow$  over-pessimistic
  - linear programming**  $\rightarrow$  **exact bounds**
- Static priorities: improvement of the existing bounds

# Collaborations

## ANR Pegase

- with M. Boyer (ONERA), E. Thierry, B. Gaujal, N. Navet, Thalès
  - Applications: wormhole routing, AFDX
- 
- 1 Univ. Pisa (G. Stea): FIFO queues [performances]
  - 2 INRIA, DistribCom (A. Benveniste): LTTA modeling [model]
  - 3 INRIA, Ipso (E. Faou) and Paris 6 (M. Zavidovique): weak KAM integrators [algorithms]

# Ongoing and future work

## Network Calculus

- Starting collaboration on stochastic network calculus and traffic modeling (Ludivic Noirie, Alcatel, LINCS)
- Network calculus for wireless networks (with Bartek)

## Perfect sampling

With Ana - submission of an ANR proposal led by A. Jean-Marie (INRIA Maestro).

## Algorithms for dynamic graphs

Master student.