What Geometry of Wireless Networks?

a few flowers from our Euclidean garden

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Inria Evaluation Seminar

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OUTLINE

- SHADOWING IN WIRELESS CELLULAR NETWORKS
  when Honeycomb is as Poisson.

- CONVEX COMPARISON OF NETWORK ARCHITECTURES
  and what if not Poisson?

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- SHADOWING IN WIRELESS CELLULAR NETWORKS
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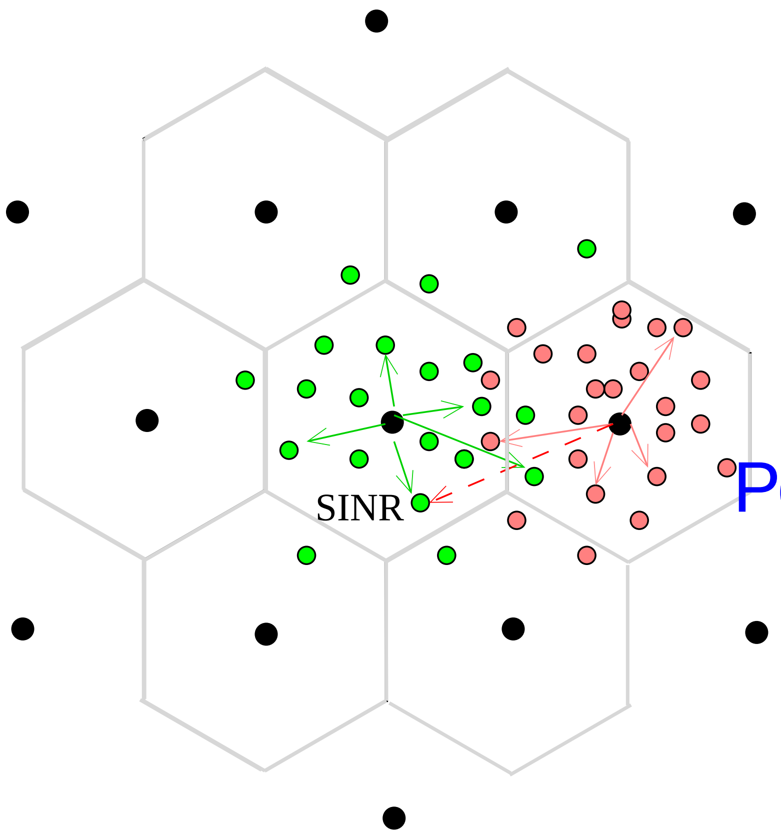
part of the research activity developed in strong industrial collaboration with M.K. Karray [Orange Labs]

- CONVEX COMPARISON OF NETWORK ARCHITECTURES
  and what if not Poisson?

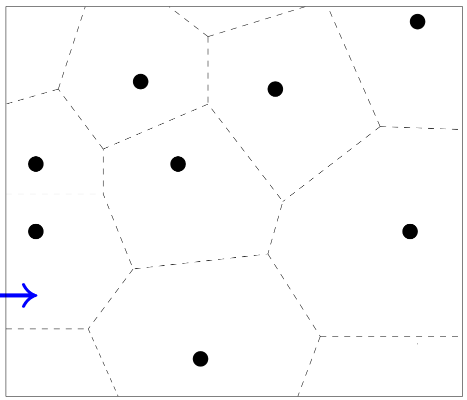
more theoretical work from PhD thesis of D. Yogeshwaran (currently Technion) founded by EADS.

SHADOWING IN WIRELESS CELLULAR NETWORKS

Cellular networks — geometry & dynamics



←Honeycomb
vs
Poisson-Voronoi→

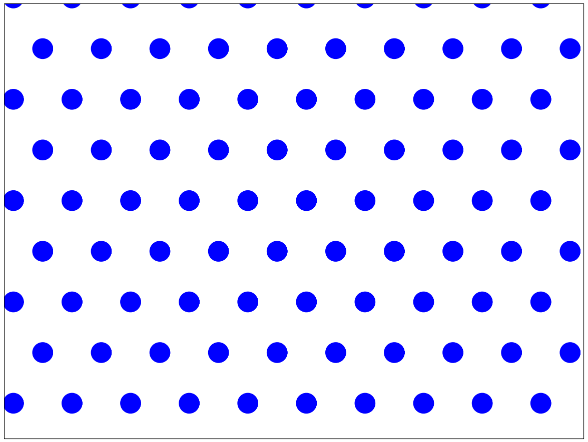


geometry: (static) pattern of BS with their path-loss fields,
dynamics: arrivals/departures/mobility of users

Questions: SINR based QoS prediction ⇒ capacity models ⇒ operator dimensioning tools.

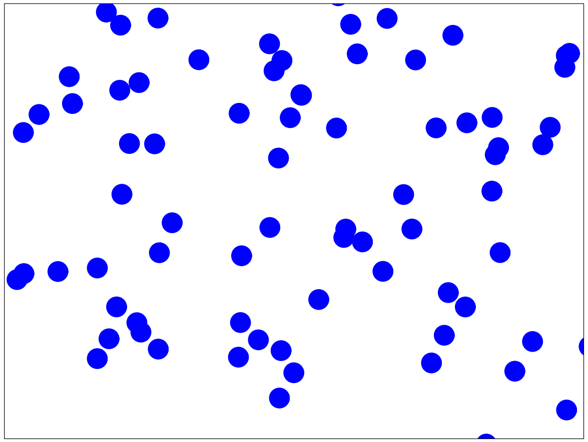
Geometry

Hexagonal



"ideal" model

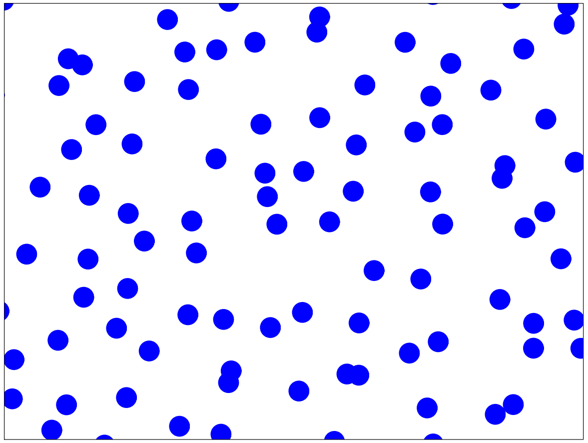
Poisson



"ad-hoc" deployed network

"Intermediate" models?

$\Rightarrow$ Convex stochastic comparisons of architectures [PhD of D. Yogeshwaran (2009)]



Dynamics

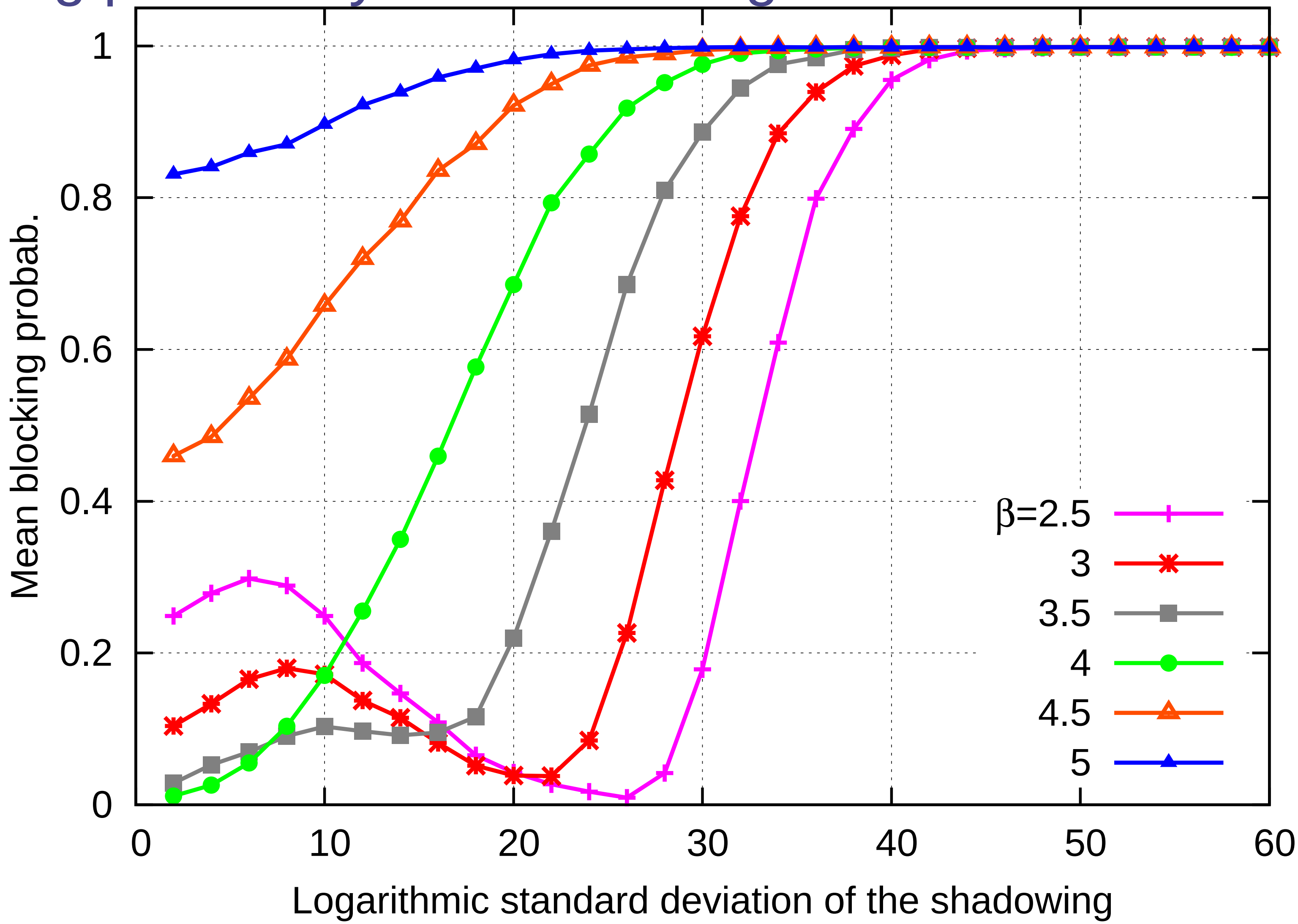
- space-time (Poisson) arrivals of calls to the network,
- demand service times at CBR (streaming), or transmission (data) volumes at VBR,
- admission and/or bit-rate allocation policies,
- QoS: blocking probabilities, mean throughout and download times, ...
- Using (existing or new) queuing/blocking/PS models in the spatial context, integrating key constraints of lower-layers (physical/coding/MAC) of the communication protocols.

Shadowing & network geometry

- Shadowing — signal power loss due to reflection, diffraction, and scattering. Modeled by random field with log-normal marginals with mean 1 and some variance.
- Impacts geometry of cellular networks:
  Serving BS $\equiv$ with smallest path-loss $\neq$the closets one.
- Problems:
  - Is believed to degrade QoS (?) $\Rightarrow$ Not always! For indoor communications?
  - How it harms the “perfect” honeycomb? $\Rightarrow$ Makes it more Poisson-like. Poisson analysis may be useful!

Shadowing; a stochastic resonances?

Blocking probability v/s shadowing variance



OFDMA, downlink, hexagonal network of 36 BS, cell radius 0.5 km; log-normal shadowing, path-loss exponent β, traffic 34.6 Erlang per km^2. [BB-Karray (2011)]

Elements of explanation

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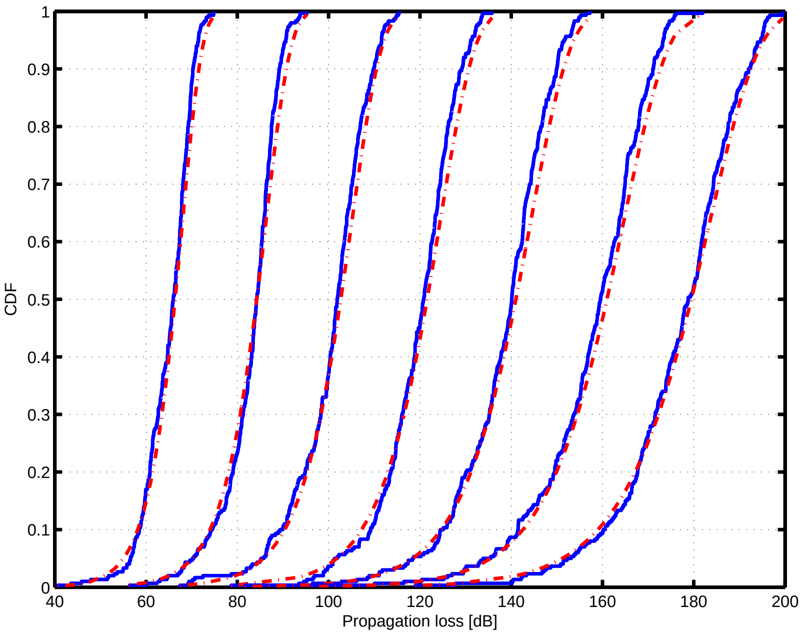
  Indeed, for log-normal S with $\mathsf{E}[S] = 1$, $1/S$ converges in Pr and in L1 to ∞, when $Var(S) \to \infty$.
- Mean interference factor (1/SIR) decreases for large variance. (Surprise?)
  Not really: for heavy tailed S_i: $\max_i S_i \sim \sum_i S_i$, hence $1/\mathsf{SIR} \sim \sum_i S_i / \max_i S_i - 1 \sim 0$.

Shadowing makes Honeycomb like Poisson

- Claim: large hexagonal network with shadowing of sufficiently large variance is perceived at a given user location as same “equivalent” infinite Poisson network (without shadowing).
- Evidences:
  - statistical testing (Kolmogorov-Smirnov) for selected metrics

    path-loss CDF
    to the serving BS⇒



  - proof of a theoretical convergence of appropriate random measures.

Application: estimation of the path-loss

- Distribution of the propagation loss $\boldsymbol{L^*}$ to the serving BS in infinite Poisson network:
$$\mathsf{P}(\boldsymbol{L^*} \geq \boldsymbol{t}) = \exp\{-\boldsymbol{\lambda\pi}\mathbf{E}[\boldsymbol{S}^{2/\beta}]\boldsymbol{t}^{2/\beta}/\boldsymbol{K}^2\}$$
where $\boldsymbol{\lambda}$ network density, $\boldsymbol{S}$ shadowing, $\boldsymbol{K}, \boldsymbol{\beta}$ path-loss constant and exponent.
- Statistics of $\boldsymbol{L^*}$ easily available from mobile measurements!
- $\Rightarrow$ Linear-regression estimation of the propagation-loss model parameters (For free for operator; no extra measurement campaigns!)

E.g. in [BB-Karray (2012)] using empirical data of Orange for Paris we estimate $\boldsymbol{\beta} = \mathbf{3.85}$, to be compared to $\boldsymbol{\beta} = \mathbf{3.80}$ obtained from (hybrid) COST Walfisch-Ikegami model.

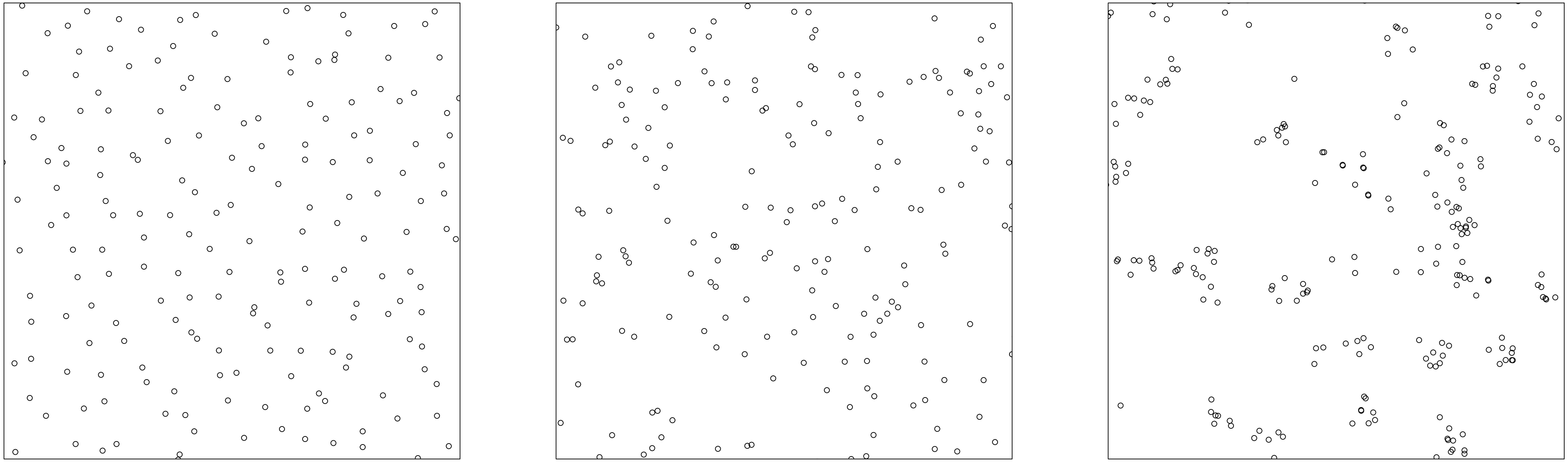
CONVEX COMPARISON OF NETWORK ARCHITECTURES

Motivation

- Ubiquitous assumptions: deterministic lattices (usually hexagonal) for BS and Poisson pp for mobile users.
- Both the assumptions are too simplistic. Observed patterns of BS are never perfectly periodic, due to various locational constraints.
  Active mobiles are not completely independent because of various interactions:
  – social, human interactions typically introduce more clustering,
  – MAC protocols tend to separate active users.
- Our goal: Develop theoretical tools for the study of impact of clustering of nodes on the performance of the networks (networks in general, and wireless in particular).

Clustering of points

Clustering in a point pattern roughly means that the points lie in clusters (groups) with the clusters being spaced out.



How to compare clustering properties of two point processes (pp) having “on average” the same number of points per unit of space?

Comparison tools

- dcx ordering of pp. Natural extension of dcx ordering of random vectors (recall Ross's conjecture), a generalization of convex ordering of random variables. Larger in dcx pp represent more variability (in probability and in state space — clustering).

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- Positive and negative association of pp. Way of comparing dependence of points to the complete independence property of Poisson pp.
- Statistical tools. Ripley function, correlation function, ... (local hence relatively weak tools).

Clustering & interference

- Result: dcx ordering of pp implies dcx ordering of the respective shot-noise fields [BB-Yogeshwaran (2009)].

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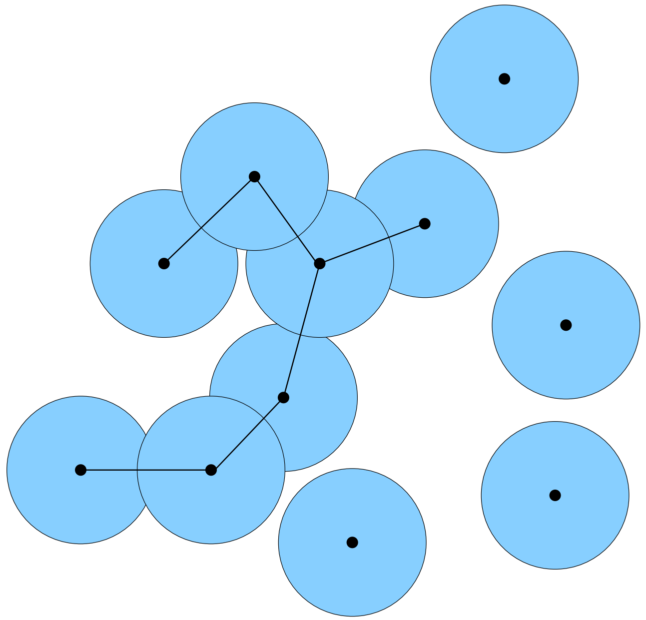
- Result: dcx ordering of pp implies dcx ordering of the respective shot-noise fields [BB-Yogeshwaran (2009)].
- "Wireless" implications: QoS metrics which are convex in interference I are improved(!) by the clustering of the pattern of interferers. (Opposite to the Ross's conjecture!)
  Examples:
  - coverage probability P$\{S/I \geq const\}$ for signal power S with convex tail distribution function (Rayleigh fading case).
  - Shannon throughput E$[\log(1 + S/I)]$.

Percolation — large scale connectivity model

Boolean model $C(\Phi, r)$:
pp Φ of antennas,
spherical grains of given radius r represent their coverage. (Interference free model.)
Joining points which are closer than r from each other one gets Random Geometric Graph (RGG).



Percolation $\equiv$ existence of an infinite connected subset (component).

Critical radius for the percolation in the Boolean Model on Φ

$$r_c(\Phi) = \inf\{r > 0 : \mathsf{P}(C(\Phi, r)\text{percolates}) > 0\}.$$

Clustering & percolation phase-transition

Result: Let Φ be a stationary pp on $\mathbb{R}^d$, weakly sub-Poisson (void probabilities and moment measures smaller than for the Poisson pp of some intensity λ). Then

$$0 < \frac{1}{(2^d\lambda(3^d-1))^{1/d}} \le r_c(\Phi) \le \frac{\sqrt{d}(\log(3^d-2))^{1/d}}{\lambda^{1/d}} < \infty;$$

[BB-Yogeshwaran (2011)].

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[BB-Yogeshwaran (2011)]. Similar results for

- k-percolation (percolation of k-covered subset) for dcx sub-Poisson.
- word percolation,
- SINR-graph percolation (graph on a shot-noise germ-grain model).

Clustering & routing

Result: Existence of stochastically too large voids in Poisson pp is the reason of infinite end-to-end packet-delivery delays in a time-space SINR model, studied in the framework of a first passage percolation problem.
[Baccelli-BB-Misradeghi (2011)]

The same problem studied on some less clustering pp (that can be shown dcx smaller than Poisson) gives finite delays.

Further results

- Monotonic in dcx models of pp based on perturbed lattices. Intended to serve as a platform for further theoretical and numerical studies of clustering.
- Comparison of voids and moments of determinantal and permanental pp, as well as all positively and negatively associated pp to these of Poisson pp.
- Counterexample of highly clustering and very well percolating pp.

FUTURE PLANS

- Clustering in random graphs (leveraging expertise of the team in the “discrete garden”)
  A prototypal result: convex ordering of the offspring degree in the Galton-Watson tree implies caparison of the extinction probabilities [from an ongoing PhD].
- QoS of streaming in LTE cellular networks [another ongoing PhD co-advised with Orange Labs].
- Network calculus for wireless network models; worst case analysis taking into account spatial constraints of the traffic (leveraging expertise of the team in the NC).
- Perfect simulations for wireless networks.